SENSITIVITY AND UNCERTAINTY IN VARIATION SIMULATION MODELS FOR MULTI-STAGE MANUFACTURING SYSTEMS

by

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INFORMATION TO USERS

To my wife, Quanbing, and my sons, William and Roy

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Variation simulation can not only help product designers with dimensional tolerance synthesis and analysis, but also provide process engineers with flexible and inexpensive means to evaluate, analyze, and diagnose a manufacturing system. The importance of the simulation in product design and process development motivated the rapid development of variation models [Shiu et al., 1996; Liu and Hu, 1996; Liu and Hu, 1997; Chang and Gossard, 1997; Jin and Shi, 1999; Mantripragada and Whitney, 1999; Lawless et al., 1999; Ding et al., 2000; and Zhou et al., 2003; Camelio et al., 2003]. This dissertation focuses on the sensitivity and uncertainty in the variation simulation models for multi-stage manufacturing systems.

Sensitivity analysis is utilized to quantify the influence of variation sources such as incoming part and manufacturing tooling variation on final product variation. Sensitivity analysis has been emphasized in manufacturing for high quality products since the robust design was proposed by Dr. Genichi Taguchi in the 1970s. Afterwards, it has been increasingly gaining attention in manufacturing. Carlson et al. [2000] pointed out that the sensitivity analysis is not only the foundation for robust design but that it can also be used for tolerance and root cause analysis. In sensitivity analysis, effective metrics play a critical and imperative role in evaluating the sensitivity of a product or system to different sources of variation.

Uncertainty is utilized to measure the difference between the model and real system or between the estimation of variables and their true values. Uncertainty is recognized to always exist in a simulation-based model for an engineering system and to strongly impact the applicability of the model. Uncertainty exists because we do not yet fully understand the real systems, or for analysis, we make many assumptions and simplifications. In other words, the uncertainties of simulation models can be caused by the errors associated with parameters of a simulation model, the errors associated with the model itself and the uncertainties of the model inputs. As shown in Figure 1-1, the simulation model uses some measured values in the physical system as its inputs. Similarly, what can be expected from the physical system are the measured values of the true outputs. Therefore, even if the simulation model captures everything of a physical system, there are still differences in the outputs between simulation models and physical systems due to measurement noises. Moreover, it is almost impossible for a simulation model to capture every aspect of a complex physical system. Hence, even if the same inputs are given to a physical system and its simulation model, different outputs are always obtained from them due to model simplifications and assumptions. All these differences contribute to the uncertainties for this simulation model.

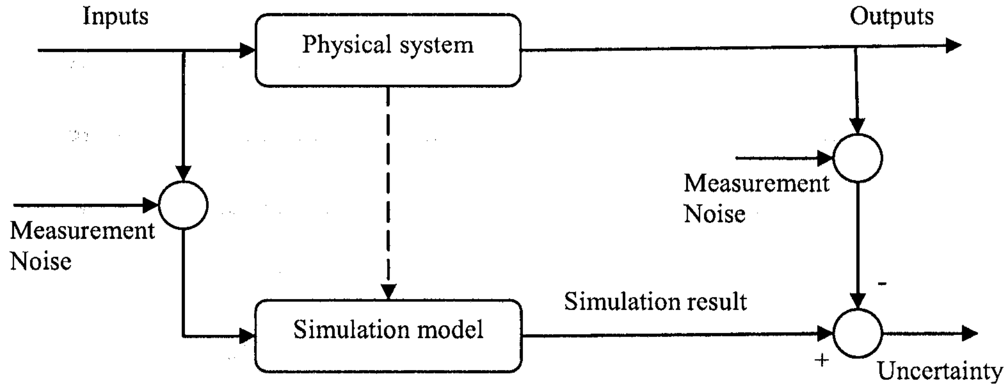

Figure 1-1. Uncertainty for the simulation model

Uncertainty incurs losses of the fidelity to the outputs of simulation models and therefore constrains the applications of the models, especially for multi-stage simulation models where uncertainty may propagate and accumulate. Consequently, the two primary issues of concern in this research are: 1) to quantify the uncertainty and to model the uncertainty propagation and accumulation in multi-stage variation simulation models, and 2) to analyze the impacts of the uncertainty on the applications of variation simulation models.

1.2 Research objectives

The objective of this research is to analyze the sensitivity and uncertainty of the variation simulation models in multi-stage manufacturing systems. The specific tasks include:

1) To define product oriented indices to measure how sensitive the final assembly dimensional quality is to the variation of a pattern, an individual part or subassembly, and the components at a stage, respectively,

2) To explore the sources and analyze the characteristics of the uncertainty for multi-stage variation simulation models,

3) To model the uncertainty propagation and accumulation in multi-stage variation simulations,

4) To apply the uncertainty model into tolerance allocation considering uncertainty of variation simulation model, and

5) To develop an algorithm to represent the complex shape of a part for good prediction and therefore low uncertainty of compliant variation simulation models at a station level.

1.3 Organization of the dissertation

This dissertation is presented in a multiple manuscript format. Each of Chapters 2, 3, 4 and 5 is written as an individual research paper, including the abstract, the main body and the references.

Chapter 2 defines three product-oriented indices which are essential for sensitivity analysis of multi-stage manufacturing systems based on the variation simulation model for compliant assemblies. These three indices include: 1) the pattern sensitivity index, the sensitivity of final assembly variation to a variation pattern; 2) the component sensitivity index, the sensitivity of final assembly variation to the dimensional variation of a component; and 3) the station sensitivity, the sensitivity of final assembly dimensional variation to the dimensional variation of all the components assembled at a particular station. Additionally, the relationship among these sensitivity indices is established and the ranges of each sensitivity index are derived [Yue et al., 2006].

Chapter 3 explores the uncertainty sources in the multi-stage variation simulation models represented in the state space form. In this chapter, uncertainty is quantified and the simulation results with uncertainty are interpreted. In addition, an uncertainty model based on multi-stage variation simulation models is developed and the uncertainty relationship between the station models and the system model is derived. The uncertainty propagation and accumulation is analyzed and the guidelines for the calibration of multi-stage variation simulation models are established using the uncertainty model [Yue et al., 2006].

Chapter 4 applies the uncertainty model into tolerance allocation considering the uncertainty of variation simulation models. The uncertainty impacts on tolerance allocation results are analyzed through comparing the proposed formulation with the traditional tolerance allocation formulation where uncertainty is not considered [Yue and Hu, 2006].

Chapter 5 shows that only the variation at clamping and joining points is not sufficient to represent the surface dimensional variation of a part /component as the inputs to compliant variation simulation models. An algorithm is proposed to decompose the surface variation of a component into the variation at the key points which are input to the variation simulation model. In order to identify the key points, a genetic algorithm (GA) is utilized [Yue et al., 2005].

Chapter 6 summarizes the conclusions and original contributions of the dissertation. Several topics are also proposed for future research.

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CHAPTER 2

PRODUCT ORIENTED SENSITIVITY ANALYSIS FOR MULTI-STAGE COMPLIANT ASSEMBLIES

Abstract

Dimensional variation in assembled products directly affects product performance. To reduce dimensional variation it is necessary that the assembly system be robust. A robust assembly process is less sensitive to incoming variation from the product and process components. In order to effectively understand the sensitivity of a system to input variation, an appropriate set of metrics must be defined. In this paper, three product oriented indices which are essential for sensitivity analysis of multi-stage compliant assembly process are defined: pattern sensitivity index, component sensitivity index and station sensitivity index. The pattern sensitivity index is the sensitivity of assembly dimensional variation to the variation patterns; the component sensitivity is the sensitivity of the assembly dimensional variation to the dimensional variation of a component; and the station sensitivity is the sensitivity of the assembly dimensional variation to the dimensional variation of all the components assembled in a particular station. Additionally, the relationships among these sensitivity indices are established, and based on these relationships, the ranges of the sensitivity indices are derived. Finally, to illustrate the applicability of these metrics, a case study of a sheet metal assembly is presented and discussed.

Nomenclature

V_w	Dimensional deviation vector for the Key Product Characteristics (KPC) of an assembly
V_u	Incoming dimensional deviation vector for the components of an assembly
X_i	State vector (n-vector) which represents the dimensional deviation of the source points at the i^{th} station in a global coordinate system
Y_i	Output vector (m-vector) for the interesting measurement points of the assembly at the i^{th} station
A_i	State matrix (n x n matrix)
B_i	Input matrix (n x r matrix)
C_i	Observation matrix (m x n matrix)
S_i	Sensitivity matrix at the i^{th} station (n x n matrix)
D_i	Deformation matrix before assembly at the i^{th} station (n x n matrix)
M_i	Re-locating matrix at the i^{th} station (n x n matrix)
U_i^{3-2-1}	Deviation vector of "3-2-1" fixtures at the i^{th} station
U_i^{n-3}	Deviation vector for the "n-2-1" ($n > 3$) fixtures at the i^{th} station
U_i^g	Deviation vector for the welding guns at the i^{th} station
W_i	Disturbance vector at the i^{th} station
V_i	Noise vector at the i^{th} station

Σ_Y	Covariance matrix of measurement points on final assemblies
Σ_{U_i}	Covariance matrix of the fixtures at the i^{th} station
Σ_{X_0}	Covariance matrix for the source points on all the incoming parts
Σ_{V_i}	Covariance matrix for the noise
$\Sigma_{X_{p\_i}}$	Covariance matrix for the source points on the i^{th} part of an assembly
$\Sigma_{X_{c\_ij}}$	Covariance matrix for the source points on the j^{th} component at the i^{th} station
λ_{ij}	the j^{th} eigenvalue of the covariance matrix $\Sigma_{X_{p\_i}}$
P_{ij}	the normalized j^{th} eigenvector of the covariance matrix $\Sigma_{X_{p\_i}}$
$\sigma^2_{Y_i}$	Variance of the i^{th} measurement point on the final assembly
m_i	Number of source points for the i^{th} part
d	Number of measurement points for the final assembly
k	Number of parts in the system
N	Number of stations in the system
ς_r, ς_1	Minimum, maximum eigenvalues of the matrix $\Upsilon(1)_i^T \Upsilon(1)_i$
η_r, η_1	Minimum, maximum singular values of the matrix $\Upsilon(1)_i$
S_{prn_ij}	Product oriented sensitivity index for the j^{th} variation pattern of the i^{th} part
S_{prt_i}	Product oriented component sensitivity index for the i^{th} part
S_{stn_i}	Product oriented station sensitivity for the i^{th} station of a system

2.1 Introduction

Multi-stage compliant assembly process is one of the most widely used in the automotive, airplane, furniture and home appliance manufacturing. The dimensional quality of assembled products plays an important role in cost, designed functionality, and customer satisfactions of the final assembly. In order to obtain high quality assemblies, several approaches and strategies have been studied. In general, these approaches fall into two categories: source variation reduction and sensitivity analysis (robust design). Higher quality in assemblies can be obviously achieved through reducing variation directly at the source. However, it was also noticed that reducing the variation at the source becomes increasingly complex, time consuming and costly as the variation diminishes. Because of that, the second approach, the sensitivity analysis in dimensional variation, was developed. High quality products can be achieved by reducing the sensitivity of the assembly dimensional variation to the source of variation instead of directly reducing the variation at the source. The importance of this method has been emphasized in manufacturing processes since it was proposed by Dr. Genichi Taguchi in the 1940s. After that, the sensitivity analysis has been increasingly gaining attention in more manufacturing areas. Carlson et al. [2000] pointed out that the sensitivity analysis is not only the foundation for robust design but that it can also be used for tolerance and root cause analysis.

The definition of effective sensitivity indices plays a critical role in the sensitivity analysis approaches. It is imperative to have effective indices to evaluate the sensitivity of a product or system to different sources of variation. In addition, the sensitivity indices can be also instructive for variation reduction techniques. For example, through these indices, the importance of each source of variation can be easily captured; and therefore, the variation reduction can be focused on the most important sources.

Several studies have been conducted to establish effective sensitivity indices from different aspects. Ting and Long [1996] conducted a sensitivity analysis for mechanisms based on a linear relationship between inputs and outputs. A boundary for the sensitivity was derived from the properties of the Rayleigh's quotient. In addition, they presented two guidelines to minimize the variation transmission. Ceglarek and Shi [1998] proposed a product joint evaluation index and a critical part determination index to evaluate how the different joints affect the robustness of a design, The proposed indices, which are based on the direct interactions between the components, can be used as an analytical tool to analyze and benchmark different designs regarding their dimensional integrity. Gao et al. [1998] defined tolerance sensitivity as the influence of individual component tolerances on the variation of a critical assembly feature or dimension. They proposed a new method for determining tolerance sensitivity using vector loop assembly tolerance models, and evaluated the derivative matrix of the equations with respect to the assembly variables. This derivative matrix was used to calculate the sensitivity matrix. Ding et al. [2002] stated that sensitivity analysis is more effective as an evaluation tool at the design stage, and that this is due to its input-independent property. Additionally, a process oriented sensitivity analysis at the system level based on a state space equation of variation propagation for multi-stage rigid assembly systems was performed. The sensitivity was defined for fixtures, stations and whole systems. These sensitivity indices, however, are applicable to *rigid* assembly processes only. Hu et al. [2003] proposed a method for evaluating the robustness of compliant assembly systems based on a variation simulation model. They defined variation transmission ratios and sensitivity indices, and analyzed the range of a predefined sensitivity index. The method, however, is applicable for *single* stations only. The purpose of this paper is to present a set of product oriented sensitivity indices for the analysis of multi-stage compliant assembly systems. Three indices are defined: the pattern sensitivity index, the component sensitivity index and the station sensitivity index. The relationship among these indices and the indices ranges are also evaluated.

The remainder of the paper is organized as follows: Section 2.2 reviews the background in multi-stage simulation model for compliant assemblies used to construct the proposed sensitivity indices. Section 2.3 introduces the three proposed product oriented sensitivity indices, and the methodology used to define them. Section 2.4 discuses a case study which illustrates how to apply the proposed indices in the different aspects of product design. Finally, conclusions are drawn in Section 2.5.

2.2 Background on multi-stage compliant assembly Modeling

To describe the dimensional relationship between an assembly and its components at the station level, Liu et al. [1996] and Liu and Hu [1997] proposed a linear model:

$$V_w = S \cdot V_u \tag{1}$$

where V_w and V_u are vectors that represent the dimensional variation of the Key Product Characteristics (KPCs) of the assembly and its components, respectively; and S

is the sensitivity matrix which can be obtained by the influence coefficient method presented in Liu and Hu [1997].

To describe the dimensional variation propagation along the stations for a compliant assembly process, Camelio et al. [2003] proposed to use a state space model:

$$
\begin{aligned}
X_i &= A_i X_{i-1} + B_i U_i + W_i \\
Y_i &= C_i X_i + V_i
\end{aligned}
\tag{2}
$$

where X_i and X_{i-1} are the state vectors; A_i is the state matrix; B_i is the input matrix; U_i is the input vector; C_i is the observation matrix; W_i is the disturbance vector; and V_i is the measurement noise vector.

The state equation in the state space model (Eq. 2) for the dimensional variation propagation for compliant assemblies can be rewritten as (Camelio et al. [2003]):

$$
\begin{aligned}
X_i = {} & (S_i - D_i + I)(X_{i-1} + M_i(X_{i-1} - U_i^{3-2-1})) \\
& -(S_i - D_i)(U_i^{n-3} + U_i^{g}) + W_i
\end{aligned}
\tag{3}
$$

where the state vector, X_i, is defined as a vector of dimensional variation including the KPCs points and Key Control Characteristics (KCCs) points for all the components at the i^{th} station.

In order to obtain A_i and B_i, the re-locating matrix M and deformation matrix D were defined and derived (Camelio et al., [2003]). The re-locating matrix explains how the state vector changes due to the change on the locating scheme from the previous station to the current station. On the other hand, the deformation matrix considers the initial shape of the parts or subassemblies.

In addition, U_i is defined as the input vector, which includes the dimensional variation of the "n-2-1" locating and holding fixtures and the welding guns. The input vector U_i can be decomposed into: locating fixtures, which are denoted as U_i^{3-2-1}; the "n-3" (n>3) additional holding fixtures, denoted as U_i^{n-3} and the dimensional variation of the assembly tools, which is denoted as U_i^g. In compliant assembly system, the assembly tools variation is usually corresponding to the welding gun variation.

With an assumption that the fixture scheme is "3-2-1" rather than "n-2-1" ($n > 3$) and welding guns are perfect, U_i^{n-3} and U_i^g are correspondingly equal to zero and, therefore, Eq. (3) can be simplified as follows:

$$\begin{aligned} X_i &= (S_i - D_i + I)(X_{i-1} + M_i(X_{i-1} - U_i^{3-2-1})) + W_i \\ &= (S_i - D_i + I)(M_i + I)X_{i-1} \\ &\quad -(S_i - D_i + I)M_i U_i^{3-2-1} + W_i \\ &= A_i X_{i-1} + B_i U_i^{3-2-1} + W_i \end{aligned} \tag{4}$$

where,

$$A_i = (S_i - D_i + I)(M_i + I)$$

$$B_i = -(S_i - D_i + I)M_i$$

Considering the sequential assembly process, the state equation and observation equation can be written as follows:

$$X_i = \Phi(i,1)X_0 + \sum_{j=1}^{i}\left(\Psi(i,j)U_j\right)$$
$$Y = \Upsilon(1)X_0 + \sum_{i=1}^{N}\Gamma(i)U_i \tag{5}$$

where,

$$\Phi(i,j) = A_i * A_{i-1} * \cdots * A_{j+1} * A_j \quad (i \geq j)$$
$$\text{and} \quad \Phi(j,j) = A_j$$
$$\Psi(i,j) = A_i * A_{i-1} * \cdots * A_{j+1} * B_j \quad (i \geq j)$$
$$\text{and} \quad \Psi(j,j) = B_j$$
$$\Upsilon(i) = C * \Phi(N,i)$$
$$\Gamma(i) = C * \Psi(N,i)$$

N is the number of the stations for the assembly system; and X_0 is the deviation vector for the source points of all the incoming parts.

The noise and disturbance effects are neglected in Eq. (5). The equation describes how the deviation of each part or subassembly propagates during the assembly process and is accumulated into the final assembly. In order to obtain the equations for the variance propagation, it is assumed that the fixtures variances are independent of each other and they are also independent of the part variances. Under this assumption, the following equation about the variances can be derived from Eq. (5):

$$\Sigma_Y = \sum_{i=1}^{N}\Gamma(i)\Sigma_{U_i}\Gamma(i)^T + \Upsilon(1)\Sigma_{X_0}\Upsilon(1)^T \tag{6}$$

where Σ_Y is the covariance matrix of measurement points on the final assembly; Σ_{U_i} is the covariance matrix of the errors in the '3-2-1' fixtures at the i^{th} station; and Σ_{X_0} is the covariance matrix for the source points on all the incoming parts.

Eq. (6) shows that the variance of final assemblies comes from two main sources: the variance of the processes (fixtures and welding guns) and the variance of incoming parts. Correspondingly, there are two types of sensitivity analysis. The first type of analysis refers to the sensitivity of the final assembly dimensional variation to the dimensional variation of the components of a process, such as fixtures and welding guns. This type of sensitivity analysis is called process oriented sensitivity analysis. The second type is about the sensitivity of the final assembly dimensional variation to its components dimensional variation, which is called product oriented sensitivity analysis. The process oriented sensitivity analysis for compliant assembly systems can be conducted by applying the sensitivity analysis methodologies for fixtures in a multi-stage rigid assembly system (Ding et al. [2002]). Product oriented sensitivity is not covered in that paper. In compliant assembly systems, component dimensional variation will alter the location of the locating fixture points, clamp points, welding points and measurement points on the components. In contrast, in a rigid assembly system, component dimensional variation only affects the final product variation through impacting the fixture location points and measurement points. Therefore, component variation will affect product variation more significantly in compliant assembly systems than in rigid assembly systems. Product oriented sensitivity analysis plays an important role in the design and analysis of compliant assembly systems. In addition, for the fixtures at a particular station, it can be reasonably assumed that the variation of the fixtures is independent of each other in the process oriented sensitivity analysis. In product oriented sensitivity for a component in compliant assembly systems, the variation of the source points on the same surface of the component is obviously dependent to each other. Therefore, the process oriented sensitivity analysis methodologies proposed by Ding et al. [2002] cannot directly apply into the product oriented sensitivity analysis for multi-stage compliant assembly systems. The remaining of this paper focuses on the methodologies of product oriented sensitivity analysis for a multi-stage compliant assembly system.

Product oriented sensitivity analyzes how the dimensional variation of final products is sensitive to the source variation of parts or components. Since it is assumed that the tooling variation is independent of the variation of parts, the tooling variation will not have any contribution to the product oriented sensitivity. Therefore, assuming that the tooling does not contribute to any variation in the assembled product, Eq. (6) can be rewritten as follows:

$$\Sigma_Y = \Upsilon(1)\Sigma_{X_0}\Upsilon(1)^T \tag{7}$$

In Eq. (7), Σ_{X_0} can be written as a block diagonal matrix by assuming that the source variation between parts is independent of each other:

$$\Sigma_{X_0} = \begin{pmatrix} \Sigma_{X_{p\_1}} & 0 & \cdots & 0 \\ 0 & \Sigma_{X_{p\_2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \Sigma_{X_{p\_k}} \end{pmatrix}$$

where $\Sigma_{X_{p\_i}}$ is the covariance matrix for the source points on the i^{th} part. The dimension of matrix $\Sigma_{X_{p\_i}}$ is $m_i \times m_i$, where m_i is the number of source points in the i^{th} part, and k is the number of incoming parts in the system. Correspondingly, the matrix $\Upsilon(1)$ can be partitioned as:

$$\Upsilon(1) = \begin{pmatrix} \Upsilon(1)_1 & \Upsilon(1)_2 & \cdots & \Upsilon(1)_k \end{pmatrix}$$

Then, Eq. (7) can be written as:

$$\begin{aligned} \Sigma_Y &= \Upsilon(1)\Sigma_{X_0}\Upsilon(1)^T \\ &= \Upsilon(1)_1\Sigma_{X_{p\_1}}\Upsilon(1)_1^T + \Upsilon(1)_2\Sigma_{X_{p\_2}}\Upsilon(1)_2^T \\ &\quad + \cdots + \Upsilon(1)_k\Sigma_{X_{p\_k}}\Upsilon(1)_k^T \\ &= \sum_{i=1}^{k}\Upsilon(1)_i\Sigma_{X_{p\_i}}\Upsilon(1)_i^T \end{aligned} \tag{8}$$

2.3 Product oriented sensitivity analysis

A typical assembly process is illustrated in Figure 2-1, Part “P1” and Part “P2” are assembled at station *1* forming the subassembly “A1”. Then, the subassembly “A1” is assembled with the Part “P3” at station *2* becoming subassembly “A2” which is then assembled with Part “P4” at station *3* to create the final assembly “A3”. At the end of the process, the final assembly is measured at the station *4*. Based on the product oriented sensitivity analysis for this system, the sensitivity of the final assembly “A3” to the variance of parts or subassemblies, “P1”, “P2”, “P3”, “P4”, “A1” or “A2”, is called component sensitivity. In addition, the sensitivity of the variance of the final assembly “A3” to the variance of “A1” and “P3”, which are the components at the second station, is called station sensitivity. Additionally, pattern sensitivity quantifies how the variance of final assemblies is sensitive to the variation patterns of a component.

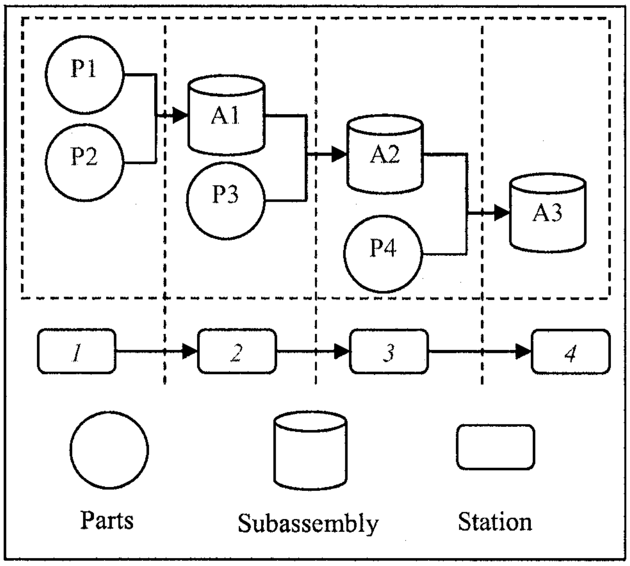

Figure 2-1. Assembly process example

2.3.1 Pattern Sensitivity

Pattern sensitivity studies the sensitivity of the assembly dimensional variation to the variation patterns of a part or subassembly. This sensitivity metric is more advantageous than a point based sensitivity to identify variation root causes. The traditional point based sensitivity is defined as the sensitivity of product dimensional variation to the variation of each source point on a part. The pattern sensitivity analysis takes the covariance information of source points into account, whereas the point based sensitivity inappropriately assumes that the source points are independent even if the points are on the same surface of a part.

The variation patterns can be obtained by applying Principal Component Analysis (PCA) on the covariance matrix of the source points on a part or component. They can also be established based on the knowledge of processes and products by engineers. Hu and Wu [1992] and Camelio et al. [2004] presented a detailed analysis of obtaining variation patterns for compliant assemblies. The importance of applying the patterns on the variation analysis is that they will often have physical interpretations of root causes for the processes or products. Therefore, the pattern sensitivity will be of interest to the designers. In other words, the designers are able to determine the impact that a common manufacturing error will have on the final product.

In order to conduct the pattern sensitivity for a part, for example, part *i*, the other parts are assumed to be perfect. Therefore, the variation propagation on Eq. (8) can be written as,

$$\begin{aligned}\Sigma_Y &= \Upsilon(1)_1 \Sigma_{X_{p\_1}} \Upsilon(1)_1^T + \Upsilon(1)_2 \Sigma_{X_{p\_2}} \Upsilon(1)_2^T \\ &\quad + \cdots + \Upsilon(1)_k \Sigma_{X_{p\_k}} \Upsilon(1)_k^T \\ &= \Upsilon(1)_i \Sigma_{X_{p\_i}} \Upsilon(1)_i^T\end{aligned} \tag{9}$$

The covariance matrix of the i^{th} part $\Sigma_{X_{p\_i}}$ can be decomposed into variation patterns using Principal Component Analysis (Camelio et al., [2004]):

$$
\begin{aligned}
\Sigma_{X_{p\_i}} &= P\Lambda P^T \\
&= \begin{pmatrix} P_{i1} \\ P_{i2} \\ \vdots \\ P_{im_i} \end{pmatrix}^T \begin{pmatrix} \lambda_{i1} & 0 & \cdots & 0 \\ 0 & \lambda_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{im_i} \end{pmatrix} \begin{pmatrix} P_{i1} \\ P_{i2} \\ \vdots \\ P_{im_i} \end{pmatrix} \\
&= \lambda_{i1} P_{i1} P_{i1}^T + \lambda_{i2} P_{i2} P_{i2}^T + \cdots + \lambda_{im_i} P_{im_i} P_{im_i}^T \\
&= \sum_{j=1}^{m_i} \lambda_{ij} P_{ij} P_{ij}^T
\end{aligned} \tag{10}
$$

Substituting Eq. (10) into Eq. (9) ,

$$
\begin{aligned}
\Sigma_Y &= \Upsilon(1)_i \Sigma_{X_{p\_i}} \Upsilon(1)_i^T \\
&= \lambda_{i1} \Upsilon(1)_i P_{i1} P_{i1}^T \Upsilon(1)_i^T + \lambda_{i2} \Upsilon(1)_i P_{i2} P_{i2}^T \Upsilon(1)_i^T \\
&\quad + \cdots + \lambda_{im_i} \Upsilon(1)_i P_{im_i} P_{im_i}^T \Upsilon(1)_i^T \\
&= \sum_{j=1}^{m_i} \lambda_{ij} \Upsilon(1)_i P_{ij} P_{ij}^T \Upsilon(1)_i^T
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\sum_{i=1}^{d} \sigma_{Y_i}^2 &= Tr\left(\Sigma_Y\right) \\
&= Tr\left(\lambda_{i1} \Upsilon(1)_i P_{i1} P_{i1}^T \Upsilon(1)_i^T\right) \\
&\quad + Tr\left(\lambda_{i2} \Upsilon(1)_i P_{i2} P_{i2}^T \Upsilon(1)_i^T\right) \\
&\quad + \cdots + Tr\left(\lambda_{im_i} \Upsilon(1)_i P_{im_i} P_{im_i}^T \Upsilon(1)_i^T\right) \\
&= \sum_{j=1}^{m_i} \lambda_{ij} Tr\left(\Upsilon(1)_i P_{ij} P_{ij}^T \Upsilon(1)_i^T\right)
\end{aligned} \tag{11}
$$

where $Tr(\cdots)$ is trace operator for a matrix.

Based on Eq. (11) and assuming that the eigenvalue λ_{ij} is equal to one and the other eigenvalues are equal to zeros, the sensitivity of the j^{th} variation pattern of the i^{th} part can be defined as follows:

$$S_{prn\_ij} = Tr\left(\Upsilon(1)_i P_{ij} P_{ij}^{T} \Upsilon(1)_i^{T} \right) \tag{12}$$

This definition measures the sum of the variance of measurement points on the final product induced by one unit variance of a particular pattern. Through this definition, the impact of most significant variation patterns of a part on the product dimensional variation can be quantified. Therefore, the importance of the root causes related to these patterns can be measured and the guidelines for product designers and process engineers to improve the quality of the product can be established.

From Eq. (12), it can be derived that:

$$\begin{aligned} S_{prn\_ij} &= Tr\left(\Upsilon(1)_i P_{ij} P_{ij}^{T} \Upsilon(1)_i^{T} \right) = \left\| \Upsilon(1)_i P_{ij} \right\|_2 \\ &= \frac{\left\langle \Upsilon(1)_i P_{ij}, \;\; \Upsilon(1)_i P_{ij} \right\rangle}{\left\langle P_{ij}, \;\; P_{ij} \right\rangle} \\ &= \frac{\left\langle \Upsilon(1)_i^{T} \Upsilon(1)_i P_{ij}, \;\; P_{ij} \right\rangle}{\left\langle P_{ij}, \;\; P_{ij} \right\rangle} \end{aligned}$$

where $\| \; \|_2$ is the operator for the Euclidean Norm of vectors. $\langle \; \rangle$ is the operator for the inner product of a pair of vectors.

Because the matrix $\Upsilon(1)_i^{T} \Upsilon(1)_i$ is a real symmetric matrix, an equation can be derived based on the property of the Rayleigh's quotient as follows:

$$\eta_r^2 = \varsigma_r \le S_{prn\_ij} \le \varsigma_1 = \eta_1^2 \qquad (13)$$

where ς_r and ς_1 are minimum and maximum eigenvalues of the matrix $\Upsilon(1)_i^T \Upsilon(1)_i$, respectively. η_r and η_1 are the corresponding minimum and maximum singular values of the matrix $\Upsilon(1)_i$.

From Eq. (13), it can be seen that the range of the sensitivity of a pattern only depends on the matrix $\Upsilon(1)_i$. Therefore, all the patterns of a part, for example, part *i*, have the same range for their sensitivity indices since the patterns have the same matrix $\Upsilon(1)_i$ which is independent of the covariance matrix of the incoming parts.

2.3.2 Component Sensitivity

Component sensitivity studies the sensitivity of the assembly dimensional variation to the variation of one of the parts/components of the assembly. The impact of each component on the final product dimensional variation can be quantified and compared through this metric. It has been also shown that the component sensitivity is the weighted sum of all the sensitivities of the component patterns. Based on this relationship and the range for the patterns sensitivity of a component, the range for the sensitivity of the component is derived in this section.

As Eq. (8) shows, all the parts of an assembly contribute to the assembly variation. In order to measure the sensitivity of the assembly variation to the variation of an individual part, for example, part *i*, it is assumed that all the other parts are perfect. Then,

$$\Sigma_Y = \sum_{i=1}^{k} \Upsilon(1)_i \Sigma_{X_{p\_i}} \Upsilon(1)_i^T = \Upsilon(1)_i \Sigma_{X_{p\_i}} \Upsilon(1)_i^T \tag{14}$$

This equation is the same as the Eq. (9). Based on this equation, the sensitivity of an individual part, for example, the i^{th} part, can be formulized as follows:

$$S_{prt\_i} = \frac{Tr(\Sigma_Y)}{Tr(\Sigma_{X_i})} = \frac{Tr\left(\gamma(1)_i \Sigma_{X_{p\_i}} (\gamma(1)_i)^T\right)}{Tr\left(\Sigma_{X_{p\_i}}\right)} \tag{15}$$

where the trace of covariance matrix of the measurement points on the final product Σ_Y represents the sum of the variance of the measurement points. The trace of the covariance matrix $\Sigma_{X_{p\_i}}$ represents the sum of the variance of the source points on the i^{th} part. This definition quantifies the joint impact of the variance of all the source points on the i^{th} part to the final product variance.

From this definition, it can be derived that

$$\begin{aligned}
S_{prt_i} &= \frac{Tr\left(\gamma(1)_i \Sigma_{X_{p_i}} (\gamma(1)_i)^T\right)}{Tr\left(\Sigma_{X_{p_i}}\right)} \\
&= \frac{\sum_{j=1}^{m_i} Tr\left(\lambda_{ij} \Upsilon(1)_i P_{ij} P_{ij}^T \Upsilon(1)_i^T\right)}{Tr\left(\Sigma_{X_{p_i}}\right)} \\
&= \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p_i}}\right)} Tr\left(\Upsilon(1)_i P_{ij} P_{ij}^T \Upsilon(1)_i^T\right) \right) \\
&= \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p_i}}\right)} S_{prn_ij} \right)
\end{aligned}$$

where λ_{ij} is the j^{th} eigenvalue of the covariance matrix $\Sigma_{X_{p\_i}}$ of the source points on the i^{th} part and it is also the variance of the pattern. S_{prn_ij} is the sensitivity index for the j^{th} variation pattern of the i^{th} part.

From the above equation, it can be seen that the sensitivity of a part is equal to the weighed sum of the sensitivities of all the patterns of the part. And the weight coefficients are:

$$Coeff_j = \frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)}, \text{ where, } j = 1, \cdots, m_i$$

Based on the eigenvalues properties of a matrix, it is known that

$$Tr\left(\Sigma_{X_{p\_i}}\right) = \sum_{j=1}^{m_i} \lambda_{ij}$$

Therefore, the sum of all the weight coefficients is equal to one.

$$\sum_{j=1}^{m_i} \left(Coeff_i\right) = \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)} \right) = 1$$

It is also known that the sensitivity of a variation pattern of a part has a range as follows:

$$\eta_r^2 \leq S_{prn\_ij} \leq \eta_1^2$$

Based on this equation and the weighted sum relationship of sensitivities between the sensitivities of the patterns of a part and the sensitivity of the part, it can be showed that

$$
\begin{aligned}
S_{prt\_i} &= \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)} S_{prn\_ij} \right) \\
&\le \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)} \eta_1^2 \right) \\
&= \eta_1^2
\end{aligned}
$$

and,

$$
\begin{aligned}
S_{prt\_i} &= \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)} S_{prn\_ij} \right) \\
&\ge \sum_{j=1}^{m_i} \left(\frac{\lambda_{ij}}{Tr\left(\Sigma_{X_{p\_i}}\right)} \eta_r^2 \right) \\
&= \eta_r^2
\end{aligned}
$$

Therefore,

$$\eta_r^2 \le S_{prt\_i} \le \eta_1^2 \tag{16}$$

Again, η_r and η_1 are the corresponding minimum and maximum singular values of the matrix $\Upsilon(1)_i$.

It must be noticed that this range is independent of the covariance matrix of incoming parts. This property is important because the component sensitivity can be estimated even when the covariance matrix is unknown, which is not uncommon, especially at the design stage.

2.3.3 Station Sensitivity

The station sensitivity analysis studies the sensitivity of the assembly variation to the variation of all the parts /components interacting in a particular assembly station. This sensitivity index quantifies the overall impact of the assembly process performed at a station to the final product dimensional variation from the product point of view. In addition, the relationship between the sensitivity of a station and the sensitivities of the components at this station is shown. Based on this relationship, the range of the sensitivity for a station is derived.

Similarly to the derivation of Eq. (6), a equation is derived as follows:

$$\Sigma_Y = \sum_{j=i}^{N} \Gamma(j)\Sigma_{U_j}\Gamma(j)^T + \Upsilon(i)\Sigma_{X_{i-1}}\Upsilon(i)^T \tag{17}$$

where $\Gamma(j)$ and $\Upsilon(i)$ were defined in Eq. (5).

This equation describes how the variance of all the fixtures and components from station i to the last station of the system propagate to the final assembly. In contrast, Eq. (6) describes how the variance of all the fixtures and components at all the stations of the system propagate to the final assembly.

Based on the assumption that the variation of fixtures and welding guns is independent of the variation of parts, the variation of fixtures and welding guns is neglected for the study of sensitivity analysis for a particular station. Therefore, Eq. (17) is written as follows:

$$\Sigma_Y = \Upsilon(i)\Sigma_{X_{i-1}}\Upsilon(i)^T \tag{18}$$

The variance relationship between the final assembly, Y, and the components after the i^{th} station inclusively, X_{i-1}, is shown in Eq. (18).

Matrix, $\Sigma_{X_{i-1}}$, can be written as a block diagonal matrix as follows:

$$\Sigma_{X_{i-1}} = \begin{bmatrix} \Sigma_{X_{c\_i1}} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ & \Sigma_{X_{c\_i2}} & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & \Sigma_{X_{c\_(i+1)1}} & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ & & & & \Sigma_{X_{c\_(i+1)2}} & 0 & \cdots & 0 & 0 & \cdots \\ & & & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & & & & \ddots & \vdots & \vdots & \vdots \\ & & & & & & & \Sigma_{X_{c\_N1}} & 0 & \cdots \\ & & & & & & & & \Sigma_{X_{c\_N2}} & \cdots \\ & & & & & & & & & \ddots \end{bmatrix}$$

where $\Sigma_{X_{c\_ij}}$ is the covariance matrix for the source points on the j^{th} component at the i^{th} station.

Correspondingly, the matrix $\Upsilon(i)$ can be partitioned as:

$$\Upsilon(i) = \begin{bmatrix} \Upsilon(i)_{c\_i1} & \Upsilon(i)_{c\_i2} & \cdots & \Upsilon(i)_{c\_(i+1)1} & \Upsilon(i)_{c\_(i+1)2} & \cdots & \Upsilon(i)_{c\_N1} & \Upsilon(i)_{c\_N2} & \cdots \end{bmatrix}$$

Then, Eq. (18) can be written as follows with the assumption that the components on the other stations except station i are perfect:

$$\begin{aligned}\Sigma_Y &= \Upsilon(i)\Sigma_{X_{i-1}}\Upsilon(i)^T \\ &= \sum_{j=i}^{N}\left(\sum_{r=1}\left(\Upsilon(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\Upsilon(i)_{c\_ir}\right)^T\right)\right) \\ &= \sum_{r=1}\left(\Upsilon(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\Upsilon(i)_{c\_ir}\right)^T\right)\end{aligned}$$

Based on this equation, the product oriented sensitivity for station i is defined as follows:

$$\begin{aligned}S_{stn\_i} &= \frac{Tr(\Sigma_Y)}{Tr\left(\Sigma_{X_{i-1}}\right)} \\ &= \frac{Tr\left(\sum_{r=1}\left(\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\gamma(i)_{c\_ir}\right)^T\right)\right)}{Tr\left(\sum_{r=1}\left(\Sigma_{X_{c\_ir}}\right)\right)}\end{aligned} \tag{19}$$

This definition for the sensitivity index shows how the variation of the components at a station will be accumulated on the final assembly. For example, if S_{stn_i} is small, it can be concluded that the component variation at the i^{th} station is diminished on the final assembly. Or else, it is increased.

From Eq. (19), it can be derived that

$$
\begin{aligned}
S_{stn\_i} &= \frac{Tr(\Sigma_Y)}{Tr(\Sigma_{X_{c\_i}})} \\
&= \frac{Tr\left(\sum_{r=1}\left(\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\gamma(i)_{c\_ir}\right)^T\right)\right)}{Tr\left(\sum_{r=1}\left(\Sigma_{X_{c\_ir}}\right)\right)} \\
&= \frac{\sum_{r=1}\left(Tr\left(\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\gamma(i)_{c\_ir}\right)^T\right)\right)}{\sum_{r=1}\left(Tr\left(\Sigma_{X_{c\_ir}}\right)\right)} \\
&= \sum_{r=1}\left(\frac{Tr\left(\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\gamma(i)_{c\_ir}\right)^T\right)}{\sum_{r=1}\left(Tr\left(\Sigma_{X_{c\_ir}}\right)\right)}\right) \\
&= \sum_{r=1}\left(\frac{Tr\left(\Sigma_{X_{c\_ir}}\right)}{\sum_{r=1}\left(Tr\left(\Sigma_{X_{c\_ir}}\right)\right)}\frac{Tr\left(\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\left(\gamma(i)_{c\_ir}\right)^T\right)}{Tr\left(\Sigma_{X_{c\_ir}}\right)}\right) \\
&= \sum_{r=1}\left(\frac{Tr\left(\Sigma_{X_{c\_ir}}\right)}{\sum_{r=1}\left(Tr\left(\Sigma_{X_{c\_ir}}\right)\right)}S_{prt\_c\_ir}\right)
\end{aligned}
$$

where $S_{prt\_c\_ir}$ is the sensitivity index for the r^{th} component at the i^{th} station.

From the preceding equation, it can be seen that the sensitivity of a station is equal to the weighted sum of the sensitivities of all the components at the station, where one of the weight coefficients is:

$$
Coeff_r = \frac{Tr\left(\Sigma_{X_{c\_ir}}\right)}{\sum_{r=1}\left(Tr\left(\Sigma_{X_{c\_ir}}\right)\right)}
$$

It can also be shown that the sum of all the weight coefficients is equal to one. Therefore, the range of the sensitivity for a station is obtained as follows:

$$
S_{prt\_c\_ir}^{\min} \leq S_{stn\_i} \leq S_{prt\_c\_ir}^{\max}
$$

where $S^{\min}_{prt\_c\_ir}$ and $S^{\max}_{prt\_c\_ir}$ are the minimum and maximum among all the sensitivities for the components at the i^{th} station.

In summary, pattern, component and station sensitivity index are defined for multi-stage compliant assembly systems from a product point of view. The relationships among these sensitivity metrics are also developed. Based on these relationships, the ranges for these sensitivities are derived which are independent of the covariance matrices of incoming parts and /or components.

From Eq. (13) and Eq. (16), it can be seen that pattern sensitivity and component sensitivity for the i^{th} part are within the range of η_r^2 and η_1^2, the minimum and maximum singular values of the matrix $\Upsilon(1)_i$. Therefore, if η_r^2 is greater than 1, all the pattern sensitivities and component sensitivity will be greater than 1. In other words, variance of the incoming patterns or the variance of the components will be amplified during the assembly process. Similarly, it can be concluded that the variance of the patterns or the variance of the component will be reduced if η_1^2 is less than 1.

In addition, the sensitivity indices for all the patterns of a particular component have the same boundary $\begin{bmatrix}\eta_r^2 & \eta_1^2\end{bmatrix}$. An index can be defined as follows:

$$index_{prn} = \frac{\eta_1^2 - \eta_r^2}{\eta_r^2}$$

Through this index, the sensitivity of the product variance to the different patterns of a particular component can be quantified. For example, if this index is small, all the patterns will have very similar potential impacts on the final product variance. The extreme case is that the unit variance of different patterns has the same impact on the final product variance if this index is equal to zero. In contrast, if this index is big, the patterns will have very different potential impacts on the final product variance. Therefore, this index can be used to evaluate the product robustness to the variances induced by different patterns and root causes.

2.4 Case study

A case study is presented in this section to illustrate the applicability of the proposed methodology in a real assembly product. The product in this case study is a side frame structure of a car. Even though the compliant assembly usually is composed of sheet metal parts and this structure consists of some sheet metal parts and some non-sheet metal parts, the proposed method can still be applied to conduct sensitivity analysis considering the deformation of the non-sheet parts only due to assembly mechanical forces. Pattern, component and station sensitivities are illustrated through this example.

Figure 2-2 shows a simplified finite element model for the side frame structure. There are seven parts in this structure. Part 1 is a 3 mm thickness hollow block. Part 2 is a hydro-formed rail and its thickness is 1 mm. Part 3 is fabricated by extrusion and its thickness is 2 mm. Parts 4, 5, 6 and 7 are stamped sheet parts and their thicknesses are 1 mm. All the parts in this assembly are made of steel.

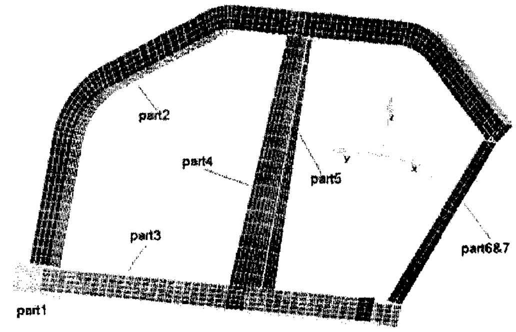

Figure 2-2. Finite element model for an automotive side frame structure

As shown in Figure 2-2, there are seven parts on this structure. These parts are welded together at three stations. Part 2 and 3 are welded to Part 1 at the first station. Part 4 and Part 5 are added at the second station. Part 6 and Part 7 are added to form the final assembly at the third station. After the assembly operations at the third station, the deviations of nine points along the whole frame are measured in Y (out of plane) and Z (up-down) directions.

At each station, the method of influence coefficient is applied to obtain the sensitivity matrix, assuming that all the welding points have some variation in both the Y and Z directions. Based on calculations of the sensitivity matrices at the station level, the state space model for this three-stage assembly system is established and the corresponding matrices are obtained.

Table 2-1. Summary of component sensitivity analysis

Part #	Y direction	Z direction	Total
Part 1	0.03	0.2	0.23
Part 2	0.6	4.4	5.0
Part 3	1.1	7.9	9.0
Part 4	0.4e-5	1.0e-5	1.4e-5
Part 5	0.3e-5	1.7e-5	2.0e-5
Part 6	0.1e-5	0.9e-5	1.0e-5
Part 7	0.1e-5	1.6e-5	1.7e-5

Based on the definition of the component sensitivity index (Eq. 15), Table 2-1 shows the results of the sensitivity analysis for the seven parts/components in the assembly. The sensitivity in the Y direction is the sensitivity of the measurement points variance in the Y and Z directions to the variance of the source points on a part in Y direction. Similarly, the sensitivity in Z direction is the sensitivity of the measurement points variance in all the Y and Z directions to the variance of the source points on a part in Z direction. The total sensitivity analysis includes the variance of both Y and Z direction on the source points of a part.

From the summary of component sensitivity analysis for each part in Table 2-1, it can be seen that the sensitivities for Part 4, 5, 6 and 7 are much smaller than those for the other three parts. One reason is that these parts are less stiff than the corresponding subassembly at their respective assembly station, which is mainly due to the geometrical structures of the parts and the assembly process requirements, such as fixture position and welding position. For instance, Part 4, 5, 6 and 7 are more compliant (flexible) than the subassembly consisting of Part 1, 2 and 3. Another reason is that the parts with small sensitivities are only involved into welding once during the assembly process. Therefore, their variation has fewer opportunities to propagate to the final assembly. For example, Part 4 and 5 are welded to become a subassembly only at the second station. In contrast, Part 2 and 3 are involved into the assembly process at all the three stations.

Since Part 1, 2 and 3 have more significant effects than the other parts on the dimensional variance of the final assembly, the component sensitivity analysis for these three parts is plotted in Figure 2-3. In this figure, the sensitivities for these three parts are grouped into the sensitivity in the Y direction, Z direction and the sum of both directions. The y axis in the plot is the logarithmic sensitivities for these parts.

As shown in Figure 2-3, the sensitivities in the Z direction are greater than those in the Y direction for all the parts. The reason is that the stiffness of these parts in the Z direction is bigger than that in the Y direction during the assembly process. For example, from the geometric structure of Part 3, the stiffness in Y direction should be the same as that in the Z direction. However, the difference in stiffness is because Part 3 is fixed by three fixtures in the Z direction and two in the Y direction.

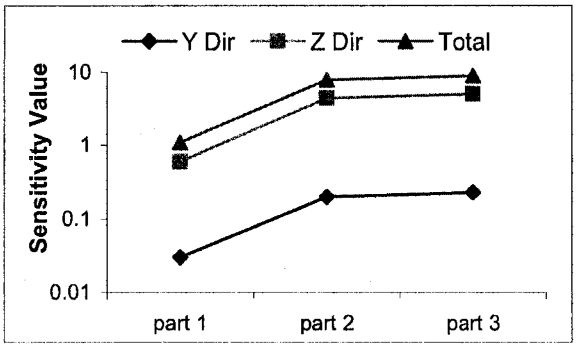

Figure 2-3. Component sensitivity for Part 1, 2 and 3

Due to the big sensitivities of Part 2 and Part 3, a sensitivity analysis for the patterns of these two parts was conducted. It is assumed that three patterns, bending about the y axis, bending about the z axis and twisting, are the major concerns for the dimensional quality of the parts. Table 2-2 summarizes the analysis results for the three patterns of each part.

Table 2-2. Sensitivity analysis for the patterns of Part 2 and Part 3

Parts	Pattern Sensitivity		
	Twisting	Bending (y)	Bending (z)
Part 2	1.02	1.9	0.3
Part 3	1.7	1.6	0.5

From Table 2-2, it can be seen that twisting and bending variation patterns about the y axis have a greater sensitivity than the bending about the z axis. This effect is reasonable because the bending pattern about z axis induces the variation in y axis in which direction Part 2 and Part 3 have small sensitivities.

Based on the proposed method, the ranges of the sensitivities for Part 2 and 3, and their patterns can be obtained. The largest and smallest eigenvalues of the corresponding $\gamma^T\gamma$ matrices for Part 2 and Part 3 are listed in the following table.

Table 2-3. Eigenvalues related to Part 2 and Part 3

Parts	Eigenvalues	
	Largest	Smallest
Part 2	5.3	0.2
Part 3	11.3	0.27

For patterns on Part 2, the sensitivities are 1.0, 1.9 and 0.3, which are within the range of (0.2, 5.3). Additionally, the component sensitivity for Part 2 is 5.0, which also is within the range of the (0.2, 5.3) as expected. Similarly, the pattern and component sensitivities of Part 3 are also within the range of (0.27, 11.3). It is proved again that the proposed method for the range of the sensitivity is valid.

The sensitivity analysis results for stations are summarized in Table 2-4. As shown in the table, it can be seen that the component variation at the second station has the most significant impacts on the dimensional variation of the final assembly. In contrast, the component variation at the first station has the least effects on the dimensional variation of the final assembly.

Table 2-4. Station sensitivity analysis

Station	Sensitivity
1	2.35
2	26.67
3	12.72

In summary, product oriented sensitivity is related to the geometry and material properties which determine the stiffness of the components and assembly process information such as fixturing schemes, welding points, and assembly sequence, and it is also related to the measurement points on the final assembly.

2.5 Conclusions

In the paper, a set of product oriented sensitivity indices has been defined for multi-stage compliant assemblies from different aspects. The component sensitivity analysis can effectively evaluate the potential contribution of a component to the product dimensional quality. The pattern sensitivity analysis for variation patterns of a part plays an important role in the process diagnosis and root cause identification due to the inherent relationship between patterns and root causes. The component sensitivity and the station sensitivity analysis evaluate the importance of a component and a station on the product variation propagation, respectively. In addition, a method has also been proposed to obtain the ranges, maximum and minimum, for all the sensitivity indices. The importance of these ranges is in that they can be used to estimate the sensitivities without any information about the incoming variation. In other words, the estimation of the sensitivities is independent of the input variation. This independence is necessary and helpful in most cases at the design stage when limited information for the components variation is available. Finally, a case study has been conducted to evaluate the definitions of these sensitivities.

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CHAPTER 3

UNCERTAINTY PROPAGATION IN VARIATION SIMULATION MODELS FOR MULTI-STAGE MANUFACTURING SYSTEMS

Abstract

Variation simulation has been widely used in product design and manufacturing system development to predict the influence of part and tooling variation on the final product quality. One of the challenges of variation simulation for multi-stage manufacturing systems is the propagation and accumulation of uncertainty, which affects the fidelity of simulation outputs. In this paper, the uncertainty sources in the multi-stage manufacturing variation simulation models represented in the state space form are explored and analyzed. The uncertainty propagation and accumulation in such models is then analyzed. Variation simulation results are explained in view of input and model parameter uncertainties. Guidelines for the calibration of multi-stage manufacturing variation simulation models are established.

3.1 Introduction

Simulation models are widely used in design and manufacturing. However, no models can completely capture all the characteristics of the simulated physical systems. It is asserted that it is impossible to specify, accurately and simultaneously, the values of the physical variables that describe the behavior of a physical system in Heisenberg

Uncertainty Principle. This inaccuracy or uncertainty characteristic of the models strongly impacts their applications, especially for multi-stage simulation models where the uncertainties can propagate and accumulate. Figure1 illustrates a multi-stage manufacturing system and its variation simulation model in the state space form. While each station is simulated using a station model, the output of one station model is the input to the next station model. For example, *X'(k-1)* represents the simulation result from station model *k-1* which is then inputted to the station model *k* along with its uncertainties. The uncertainties associated with *X'(k-1)* are consequently input to the station model *k* and introduce some uncertainties to *X'(k),* the output of the station model *k*. This uncertainty propagation and accumulation can obviously reduce the fidelity of the final simulation results. Therefore, it is important to understand this propagation and accumulation so that guidelines for simulation result interpretation and model calibration can be established.

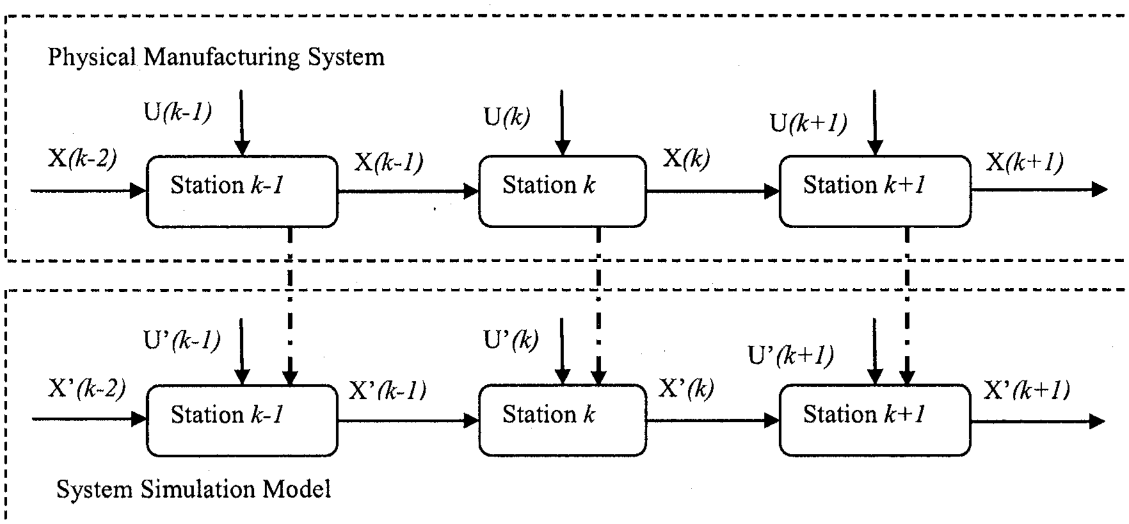

Figure 3-1. A manufacturing system and its simulation model

The reminder of this paper is organized as follows: Section 3.2 reviews uncertainty in terms of its definitions, sources and mathematical models, as well as variation simulation models for multi-stage manufacturing systems. Section 3.3 explores the uncertainty sources especially for the variation simulation models, develops an uncertainty propagation model and illustrates the propagation of uncertainties in the variation simulation for a multi-stage manufacturing system. In Section 3.4, a case study is conducted to demonstrate the proposed uncertainty model and methodologies. Section 3.5 applies the uncertainty model in the calibrations of simulation models for multi-stage manufacturing systems. Finally, Section 3.6 summarizes and concludes the paper.

3.2 Literature review

This section reviews the definitions, sources and mathematical models for uncertainties, the uncertainty in simulation models, and the variation simulation models for multi-stage manufacturing systems.

3.2.1 Uncertainty

3.2.1.1 Definitions, sources and mathematical models. Uncertainty means different things to different people. For example, Figliola and Beasley [1991] referred the error estimate for a measurement as uncertainty. Yen and Tung [1993] attributed uncertainty mainly to a lack of perfect understanding with regard to phenomena or processes. Ayyub and Gupta [1994] characterized uncertainty as an inseparable companion of any measurement at the experiential level, and as the vagueness and incompleteness of understanding of complex real problems at the cognitive level. Zhao et al. [1995] defined uncertainty as the differences or errors between models and the reality. Oberkampf et al. [1999] described uncertainty as a potential deficiency in any phase or activity of a modeling process due to a lack of knowledge. Delaurentis and Mavris [2000] provided the definition of uncertainty as incompleteness in knowledge (either in information or context) which causes model-based predictions to differ from the reality in a manner described by some distribution functions. Zimmermann [2001] defined stochastic uncertainty as the unknown of the future state of a system due to lack of information and fuzziness uncertainty as the vagueness concerning the description of the semantic meaning of events, phenomena, or statements themselves.

Uncertainty can also be classified differently from the view point of uncertainty sources as follows:

1) Natural uncertainty, also referred to as inherent uncertainty and physical randomness, which is due to the physical variability of a system [Yen and Tung, 1993; Hazelrigg, 1996; Wersching and Wu, 1996; Ayyub and Chao, 1997];

2) Model uncertainty due to simplifying assumptions in analytical and prediction models, simplified methods, and idealizing representations of real performances [Yen and Tung, 1993; Wersching and Wu, 1996; Ayyub and Chao, 1997; Gu, 1998; Du and Chen, 2000; Delaurentis and Mavris, 2000];

3) Measurement uncertainty resulting from the limitation of measurement methodologies and the capability of measurement systems [Yen and Tung, 1993; Hazelrigg, 1996; Delaurentis and Mavris, 2000];

4) Operational and environment uncertainty [Yen and Tung, 1993; Delaurentis and Mavris, 2000];

5) Statistical uncertainty due to the incompleteness of statistical data and the use of sampled information to estimate the characteristics of these parameters [Wersching and Wu, 1996; Ayyub and Chao, 1997], and

6) Subjective uncertainty related to expert-based parameter selection, human factors in calculation, fabrication and judgment [Wersching and Wu, 1996; Ayyub and Chao, 1997].

The model uncertainty was further classified as follows: (1) input uncertainty, also referred to as input parameter uncertainty, external uncertainty and precision uncertainty [Gu, 1998; Du and Chen, 2000]; (2) bias uncertainty which is induced in transforming the physical principles of scientific theory into analytic or raw models for engineering use and transforming the analytic or raw models into numerical simulation models [Gu, 1998]; (3) model parameter uncertainty arising from the limited information in estimating the characteristics of model parameters [Manners, 1990; Ayyub and Chao, 1997]; and (4) model structure uncertainty [Apostolakis, 1994; Laskey, 1996] which is due to the assumption and simplification about the model structure.

In order to incorporate and address uncertainties, several mathematical models were proposed. Examples of these models include interval model, convex model, fuzzy sets, and random model. Interval model, introduced in the early 1900s, can give rigorous bounds for a solution and was applied to different fields [Lew et al., 1994; Moore, 1966; Simoff, 1996] [Chen and Ward, 1997; Kubota et al., 1999; Penmetsa and Grandhi, 2002; Chen et al., 2004]. Convex model, extended the interval model from one dimension to multi-dimension, have been used in construction engineering, mechanical engineering, structural engineering, mechanics and other fields [Lindberg, 1992; Ben-Haim, 1994, 1996, 1997; Attoh-Okine, 2002]. Fuzzy sets, introduced by Zadeh in 1965, were initially used in fields such as economics, social sciences to address the uncertainties induced by the imprecise and vague information. Afterwards, they were extended into engineering areas [Wood and Antonsson, 1989; Wood et al., 1989; Wood et al., 1992; Antonsson and Otto, 1995]. Random model, using probability mass function or probability density function to represent the uncertainty, also has considerable applications. In addition to these models, some other models were recently introduced and applied for uncertainties [Pawlak, 1985; Deng, 1989].

3.2.1.2 Uncertainty analysis for simulation models. Uncertainty analysis for simulation models is gaining increasing attention recently due to the wide spread of computer simulations in engineering systems and the importance of the simulation results to decision makers and system designers. Several approaches, starting with the employment of safety factors in deterministic analysis, were developed. The common method used for uncertainty analysis is the sensitivity-based approximation approach that includes the worst case analysis and the moment matching method. Meyn [2000] proposed a methodology where the uncertainty in each measurement is represented as a vector of elemental uncertainties which is used to calculate the final uncertainty. Du and Chen [2001] applied MPP (Most Probable Point) based uncertainty analysis to capture the probabilistic distribution of the system output with the uncertainty in the system input. The MPP concept was utilized to generate the cumulative distribution function for system output by evaluating probability estimates for a serial of limit states across a range of output performance. In addition, a novel MPP search algorithm was presented to improve the efficiency of this method. Putko [2001] presented an implementation of the approximate statistical moment method for uncertainty propagation and robust optimization for a quasi 1-D Euler CFD code. In his analysis of uncertainty propagation, only external uncertainty (the uncertainty of input parameters) was considered. The author assumed the input variables were statistically independent, random and normally distributed about their mean values. Uncertainty propagation was accomplished by approximate statistical second moment methods where the sensitive derivation was required. Du and Chen [2002] developed System Uncertainty Analysis (SUA) and Concurrent Sub-System Uncertainty Analysis (CSSUA) methods to handle the uncertainty in multidisciplinary robust design. In essence, both of these methods were derived from the sensitivity-uncertainty idea.

Previous research has not shown how uncertainty associated with the station model propagates in a multi-stage simulation model and affects the accuracy of the final simulation results. The purpose of this paper is to present a methodology to determine how uncertainty propagates and accumulates in simulating multi-stage manufacturing systems. The methodology is based on the uncertainty model derived from a state space model.

3.2.2 Variation simulation models for manufacturing systems

This section will review the variation simulation models in the state space form which are widely used in the dimensional variation analysis for assembly and machining systems.

3.2.2.1 Variation simulation models for rigid assembly systems. In general, variation simulation deals with the problem of finding the cumulative variation of a final product given the variation of its components. The most common used variation simulation methods include Worst Case, Root Sum Square (RSS) method and Monte Carlo Simulation. A detailed review was given by Chase and Parkinson [1991], and Juster [1992]. However, both Worst Case and RSS methods are difficult to be applied to complex two or three dimensional assemblies. Monte-Carlo Simulation method can be applied to more complex assemblies along with a mathematical model to describe how the parts are assembled [Craig, 1989; Early and Thompson, 1989]. However, Monte Carlo Simulation is a sample-based method. Chase and Greenwood [1988] stated that inclusion of realistic physical/functional models of integrated product and manufacturing processes is especially important for the current technology of manufacturing complex products.

Mantripragada and Whitney [1999] distinguished two types of assemblies in the modeling of assembly processes. In comparison with Type-1 assemblies where the mating features are pre-fabricated, Type-2 assemblies can incorporate some adjustments using contact features between parts in the process. For both types of assemblies, a state transition equation was developed for the variation propagation in mechanical assemblies.

Jin and Shi [1999] proposed using a state space model for the variation propagation in multi-stage 2-D rigid body assembly systems. The model was written as:

$$X(k) = A(k)X(k-1) + B(k)U(k) + v(k) \quad (1)$$

where $X(k)$ and $X(k-1)$ were defined as vectors of accumulated deviation at the k^{th} and the $(k-1)^{th}$ station, respectively. $A(k)$ describes the relocation effects to the (sub)assembly variation from the $(k-1)^{th}$ station to the k^{th} station. $U(k)$ represents the deviation of the fixtures at the k^{th} station. $B(k)$ describes the impacts of the fixtures to the (sub)assembly deviations at the k^{th} station. $v(k)$ was defined as a disturbance vector.

This model can be used to predict the deviation of an assembly given the deviations of the incoming parts and the fixtures at all the stations. This model was developed based on three assumptions: (1) parts and subassemblies have only in-plane deviation, (2) the deviations are small compared with the dimensions of parts and therefore the system is considered linear, and (3) parts joint types are lap-joint.

3.2.2.2 Variation simulation models for compliant assembly systems. All the models reviewed above did not consider the deformation of parts during assembly, which is reasonable when the stiffness of the components in a product is relatively high. However, deformation commonly happens in the assembly process of sheet metal parts. Moreover, the assembly deviation due to the deformation sometimes may be dominant compared with the deviations due to other sources such as the homogeneous transformation resulting from misalignment or wear-out of fixtures and thermal distortions. In order to describe the dimensional relationship between a compliant assembly and its components at station level, Liu et al. [1996] and Liu and Hu [1997] proposed a linear model:

$$V_w = S \cdot V_u \tag{2}$$

where V_w and V_u are vectors that represent the dimensional variation of the Key Product Characteristics (KPCs) of an assembly and its components, respectively; and S is the sensitivity matrix considering the deformations and springbacks of products in assembly processes. It can be obtained by the influence coefficient method for complex products.

Based on the station model shown as Eq. (2), Camelio et al. [2003] extended the state space model expressed in Eq. (1) to include part compliance in order to describe the dimensional deviation propagation along the stations for a compliant assembly process. The state space model was rewritten for the dimensional deviation propagation in compliant assemblies as:

$$\begin{aligned} X(k) = &\left(S(k) - D(k) + I\right)\left(X(k-1) + M(k)\left(X(k-1) - U(k)_{3-2-1}\right)\right) \\ &-\left(S(k) - D(k)\right)\left(U(k)_{n-3} + U(k)_g\right) + v(k) \end{aligned} \tag{3}$$

where $S(k)$ is sensitivity matrix which describes the induced (sub)assembly deviation due to a unit deviation of the incoming parts at the k^{th} station. I is an identity matrix. The re-locating matrix, $M(k)$, explains how the state vector changes due to the change of the locating scheme from the previous station to the current station. On the other hand, the deformation matrix, $D(k)$, considers the initial shape of incoming parts or subassemblies. $U(k)_{3-2-1}$ was defined as the deviation of the "3-2-1" fixtures at the k^{th} station. $U(k)_{n-3}$ was defined as the deviation of the "$n-2-1(n>3)$" fixtures at the k^{th} station. $U(k)_g$ was defined as the deviation of the assembly tools at the k^{th} station.

Assuming that the fixture scheme is "3-2-1" rather than "$n-2-1(n>3)$" and assembly tools are perfect, $U(k)_{n-3}$ and $U(k)_g$ are zero and, therefore, Eq. (3) can be simplified as follows:

$$\begin{aligned} X(k) &= \left(S(k)-D(k)+I\right)\left(X(k-1)+M(k)\left(X(k-1)-U(k)_{3-2-1}\right)\right)+v(k) \\ &= \left(S(k)-D(k)+I\right)\left(I+M(k)\right)X(k-1)-\left(S(k)-D(k)+I\right)M(k)U(k)_{3-2-1}+v(k) \\ &= A(k)X(k-1)+B(k)U(k)+v(k) \end{aligned} \tag{4}$$

where,

$$A(k)=\left(S(k)-D(k)+I\right)\left(I+M(k)\right)$$

$$B(k)=-\left(S(k)-D(k)+I\right)M(k)$$

$U(k)$ is equal to $U(k)_{3-2-1}$

3.2.2.3 Variation simulation models for machining systems. Zhou et al. [2003] developed a state space model to describe the geometric error accumulation and transformation when workpieces passed through the stations of a machining system. In the model the state vector was represented by a stack of differential motion vectors of all the key features of workpieces. The input vector was represented as deviations of a tool path from its nominal path. Two assumptions were implied in the development of this state space model:

(1) the machining error on a single stage was modeled as the tool path deviation from its nominal path; and (2) only position and orientation errors were considered and profile errors were not included in the modeling.

3.2.2.4 Summary. From the reviews of the variation simulation models for rigid assembly, compliant assembly and machining systems, it can be seen that state space models are playing an important role in the dimensional quality analysis of manufacturing systems. Considering the sequential process of manufacturing systems and the different definitions for the state matrix, the state vector, the input matrix and the input vector, all the state space models proposed for assembly and machining systems can be written as follows:

$$X(k) = \Phi(k,1)X(0) + \sum_{j=1}^{k}\left(\Psi(k,j)U(j)\right) \tag{5}$$

where,

$$\begin{aligned}
&\Phi(k,j) = A(k) * A(k-1) * \cdots * A(j+1) * A(j) \quad (k \geq j) \\
&\qquad \text{and} \quad \Phi(j,j) = A(j) \\
&\Psi(k,j) = A(k) * A(k-1) * \cdots * A(j+1) * B(j) \qquad (k \geq j) \\
&\qquad \text{and} \quad \Psi(j,j) = B(j)
\end{aligned}$$

$X(0)$ is the deviation vector for the source points, including Key Product Characteristics (KPC) and/or Key Control Characteristics (KCC) points for all the incoming parts. Neglecting noise and disturbance effects, Eq. (5) describes how the deviation of each part or subassembly propagates and accumulates into the final product during manufacturing processes. In order to obtain the equations for the variance propagation, it is assumed that the different sources of variation are independent of each other. For instance, the incoming part variation is independent of the fixture variation. Under this assumption, the following equation about the variances can be derived from Eq. (5):

$$\begin{aligned}\Sigma_{X(k)} &= \Phi(k,1)\Sigma_{X(0)}\Phi^T(k,1)+\sum_{j=1}^{k}\left(\Psi(k,j)\Sigma_{U(j)}\Psi^T(k,j)\right)\\ &= \gamma(0)\Sigma_{X(0)}\gamma^T(0)+\sum_{j=1}^{k}\left(\gamma(j)\Sigma_{U(j)}\gamma^T(j)\right)\end{aligned} \tag{6}$$

where,

$$\begin{aligned}\gamma(j) &= A(k)A(k-1)\cdots A(j+1)B(j) \qquad (j=0,1\cdots,k)\\ B(0) &= I\end{aligned}$$

$\Sigma_{X(k)}$ is the covariance matrix of the deviation of the points in the state vector $X(k)$; $\Sigma_{U(j)}$ is the covariance matrix of the errors of the fixtures at the j^{th} station; and $\Sigma_{X(0)}$ is the covariance matrix for the source points on all the incoming parts.

3.3 Uncertainty in variation simulation models for multi-stage manufacturing systems

This section analyzes the sources and characteristics of uncertainty, proposes a measure of uncertainty, and develops a model for the uncertainty propagation in simulation models for a multi-stage manufacturing system.

3.3.1 Uncertainty sources

Based on the reviews in Section 3.2.1, the uncertainty sources and characteristics for the variation simulation models of a multi-stage manufacturing system are analyzed and discussed. As shown in Figure 3-1, a typical variation simulation model for a multi-stage manufacturing system consists of several station models to simulate each station of the system. In the figure, it can be seen that the uncertainties associated with the variation simulation model include the input uncertainty, the station model uncertainty, the propagated uncertainty, and the system model uncertainty.

The input uncertainty is associated with the input variables of the model. For instance, the uncertainty associated with the dimensions of the incoming parts, the fixtures and the assembly/machining tools in the system.

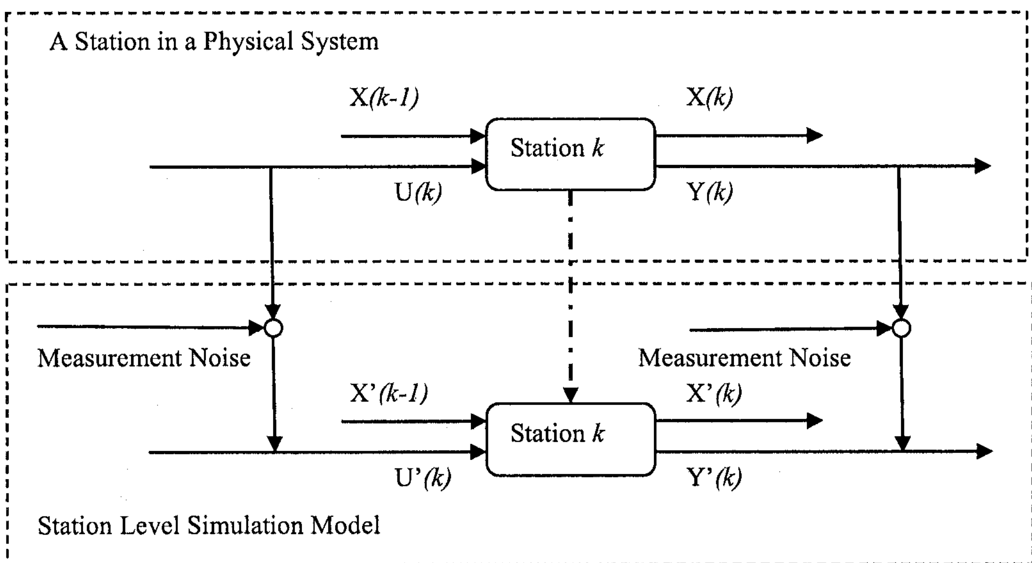

Figure 3-2. Uncertainty analysis for the station model

Figure 3-2 illustrates the station model uncertainty, where *Y(k)* and *Y'(k)* are the outputs for the measurement points at the k^{th} station for a physical systems and its corresponding simulation model, respectively. It can be seen that there are some uncertainties associated with the state vector *X(k-1)* and the input vector *U(k)* which are the inputs to the station model *k*. In addition, the state vector *X(k)* coming out of station model *k* has some uncertainties which is likewise the one of the uncertainties input to station model *k+1*. There are also some uncertainties associated with the output vector *Y(k)* which is due to measurement noises as shown in the Figure This kind of uncertainty is called observation uncertainty which can also be found in a multi-stage simulation model even it is not shown in Figure 3-1.

The propagated uncertainty is the uncertainty that circulates within the system. For example, in Figure 3-1, the uncertainty associated with *X*(*k-1*), the output of station model *k-1*, is passed to station model *k* as the input uncertainty.

The system model uncertainty is the uncertainty induced by the assumptions, simplifications and other factors during the system model extraction and establishment for the whole manufacturing system.

In summary, the uncertainties in the variation simulation models for a multi-stage manufacturing system can be classified into input, propagated, observation, and model uncertainty including both the system model uncertainty and state model uncertainty. In addition, the model uncertainty includes parameter uncertainty, the uncertainty associated with model parameters, and model structure uncertainty, the uncertainty associated with model structures.

The sources for these uncertainties can be explored and classified as:

1) Measurement error. It induces some uncertainties in the input and the observation as shown in Figure 3-2.

2) Assumptions and simplifications. Making assumptions is to ignore some unknown factors. On the other hand, simplification is to ignore some known but unwanted factors of a physical system. Because it is nearly impossible to completely understand a complex physical system, assumptions have to be made during the establishment of a simulation model. For instance, in the state space model proposed by Jin and Shi [1999] and Ding et al. [2000], the assumption of a linear relationship between the output and the input was made even the physical system may be nonlinear. These assumptions definitely induce some uncertainties. Moreover, simplifications are made to model complicated physical systems. For instance, the operator in a manufacturing system is an important factor for the system output besides the factors included in the model. However, in order to simplify the physical system, this factor is ignored, which definitely induces some uncertainties in the model. As a result, both assumptions and simplifications induce model uncertainty.

3) Propagation. In a multi-stage simulation model, the simulation results from the previous station model along with their uncertainties will be input to the current station model. Similarly, the uncertainties associated with the simulation results from the current station model will be one source of uncertainty for the next station model. Therefore, uncertainty

propagates and accumulates on the final system outputs. This uncertainty propagation is illustrated in Figure 3-1 through state vectors.

4) Computational errors. A simulation model is usually implemented on computers which have some unavoidable computational errors. This uncertainty source is not considered in this research.

5) Other errors. For example, FEM is sometimes used to model a real entity. Obviously, difference exists between the FEM model and the real entity, which can cause some input and output uncertainties. It is another uncertainty source which is not considered in this research.

The proposed methodology will focus on the input uncertainty, the model uncertainty and their propagations in a multi-stage simulation model. For the model uncertainty, only the parameter uncertainty will be discussed in this research. However, the other types of uncertainty could also be analyzed using the proposed methodology.

3.3.2 Uncertainty quantification

In order to quantify the uncertainty associated with the inputs, parameters and the outputs of a model, a formula is proposed as follows:

$$\zeta = \frac{\|Y' - Y\|_2}{\|Y\|_2} = \frac{\|\Delta Y\|_2}{\|Y\|_2} \tag{7}$$

where ζ is the uncertainty associated with a variable. Y' and Y are the values with and without considering uncertainty of the variable being studied, which can be the input variable or the parameters of a model. Y' and Y can be scalars or vectors. ΔY is the difference between the values of the variable with and without considering uncertainty. $\| \|_2$ is the second norm operator.

If it is assumed that the value of Y' is following a certain distribution, for example the uniform distribution, the distribution can be approximately obtained as:

$$Y' \sim U\left(Y - \Delta Y, \quad Y + \Delta Y\right) \tag{8}$$

From this equation, it can be seen that a model with uncertainty provides a range or a random value with a statistical distribution instead of a deterministic value as the prediction. For example, if Y', Y and ΔY are scalars instead of vectors and denoted as y', y and Δy, respectively, Eq. (8) can be written as follows:

$$y' \sim U\left(y - \Delta y \quad y + \Delta y\right) = U\left((1-\zeta)y \quad (1+\zeta)y\right) \tag{9}$$

As shown by this example, the uncertainty ζ plays an important role in the width of the range and therefore the confidence of the prediction.

3.3.3 Uncertainty model

With the knowledge of the sources and the definition for uncertainties, an uncertainty model is derived from the state space model described in Eq. (1).

Assuming that there is no model structure uncertainty, and therefore the model considering uncertainty is still a linear model, an equation can be derived from Eq. (1) as follows:

$$X'(k) = A'(k)X'(k-1) + B'(k)U'(k) \tag{10}$$

where:

$X'(k)$: state vector with uncertainty

$A'(k)$: state matrix with uncertainty

$B'(k)$: input matrix with uncertainty

$U'(k)$: input vector with uncertainty

The uncertainty associated with the state matrix A can be obtained using Eq. (7) as follows:

$$\zeta_A = \frac{\|A' - A\|_2}{\|A\|_2} = \frac{\|\Delta A\|_2}{\|A\|_2} \tag{11}$$

If it is assumed that A and ΔA have the same structure, it can be obtained as:

$$\Delta A = \zeta_A A \tag{12}$$

From Eq. (8) and Eq. (11), it can be derived that

$$A' \sim U\left(A - \Delta A,\quad A + \Delta A\right) = U\left(\left(1 - \zeta_A\right)A,\quad \left(1 + \zeta_A\right)A\right)$$

where A' is the value of the state matrix considering uncertainty.

Similarly,

$$B' \sim U\left(B-\Delta B,\quad B+\Delta B\right)=U\left(\left(1-\zeta_B\right)B,\quad \left(1+\zeta_B\right)B\right)$$

$$X'(0) \sim U\left(X(0)-\Delta X(0),\quad X(0)+\Delta X(0)\right)=U\left(\left(1-\zeta_X\right)X(0),\quad \left(1+\zeta_X\right)X(0)\right)$$

$$U' \sim U\left(U-\Delta U,\quad U+\Delta U\right)=U\left(\left(1-\zeta_U\right)U,\quad \left(1+\zeta_U\right)U\right)$$

Based on the uncertainty classification in Section 3.3.1, ζ_X, the uncertainty for incoming parts, and ζ_U, the uncertainty for fixtures, are called input uncertainty. ζ_A and ζ_B associated with the model parameters A and B are called parameter uncertainty, part of model uncertainty. With an assumption that all the parameter and input uncertainties, ζ_A, ζ_B, ζ_X and ζ_U, are equal to each other, the above equations can be rewritten as:

$$A' \sim U\left(\left(1-\zeta\right)A,\quad \left(1+\zeta\right)A\right)$$

$$B' \sim U\left(\left(1-\zeta\right)B,\quad \left(1+\zeta\right)B\right)$$

$$X'(0) \sim U\left(\left(1-\zeta\right)X(0),\quad \left(1+\zeta\right)X(0)\right)$$

$$U' \sim U\left(\left(1-\zeta\right)U,\quad \left(1+\zeta\right)U\right)$$

In addition, similar to the derivation of Eq. (6), the following equation can be derived from Eq. (10):

$$X'(k)=\gamma'(0)X'(0)+\sum_{j=1}^{k}\left(\gamma'(j)U'(j)\right) \tag{13}$$

where, $X^{'}(k)$, as defined before, is the output state vector of the k^{th} station considering uncertainty. $U^{'}(i)$ is the input vector at the i^{th} station considering uncertainty. $\gamma'(j) = A'(k)A'(k-1)\cdots A'(j+1)B'(j) \quad (j = 0,1\cdots,k;\ B'(0) = I)$, similar to the definition of $\gamma(j)$ in Eq. (6).

Since all the parameters and inputs in Eq. (13) are random variables, linearization of Eq. (13) using Taylor expansion gives the result as follows:

$$
\begin{aligned}
X'(k) &= \gamma'(0)X'(0) + \sum_{j=1}^{k}\left(\gamma'(j)U'(j)\right) \\
&\approx \gamma(0)X(0) + \sum_{i=1}^{k}\left(\Omega(i,1)X(0)\right) + \gamma(0)\left(X'(0) - X(0)\right) \\
&\quad + \sum_{i=1}^{k}\left(\gamma(i)U(i)\right) + \sum_{i=1}^{k-1}\left(\left(\sum_{m=i+1}^{k}\left(\Omega(m,i+1)\right)\right)B(i)U(i)\right) \\
&\quad + \sum_{i=1}^{k}\left(\Gamma(i)U(i)\right) + \sum_{i=1}^{k}\left(\Upsilon(i)\left(U'(i) - U(i)\right)\right) \\
&= \gamma(0)X(0) + \sum_{i=1}^{k}\left(\gamma(i)U(i)\right) \\
&\quad + k\zeta\Upsilon(0)X(0) + \xi\Upsilon(0)X(0) + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)\Upsilon(i)U(i)\right) \\
&= X(k) + \left((k+1)\zeta\right)\Upsilon(0)X(0) + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)\Upsilon(i)U(i)\right)
\end{aligned}
\tag{14}
$$

where,

$$
\begin{aligned}
\Omega(j,p) &= A(k)A(k-1)\cdots A(j+1)\left(A'(j) - A(j)\right)A(j-1)\cdots A(p) \\
&\qquad (k \geq j \geq p) \\
\Gamma(i) &= A(k)A(k-1)\cdots A(i+1)\left(B'(i) - B(i)\right) \quad (i = 1,2,\cdots,k-1) \\
\Gamma(k) &= \left(B'(k) - B(k)\right)
\end{aligned}
$$

With the assumptions stated before, the covariance relationships of the entries in state vectors can be derived from Eq. (14) as follows:

$$\Sigma_{X'(k)} \approx \Sigma_{X(k)} + \left((k+1)\zeta\right)^2 \Upsilon(0)\Sigma_{X(0)}\left(\Upsilon(0)\right)^T + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)^2 \Upsilon(i)\Sigma_{U(i)}\left(\Upsilon(i)\right)^T\right) \tag{15}$$

where, $\Sigma_{X'(k)}$, $\Sigma_{X(0)}$ and $\Sigma_{U(i)}$ are the covariance matrix of $X'(k)$, $X(0)$ and the fixtures at the i^{th} station $U(i)$, respectively.

From Eq. (15), the difference of the covariance matrices for the entries in the state vector at the k^{th} station with and without considering uncertainties can be obtained as follows:

$$\Delta\Sigma_{X(k)} \approx \left((k+1)\zeta\right)^2 \Upsilon(0)\Sigma_{X(0)}\left(\Upsilon(0)\right)^T + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)^2 \Upsilon(i)\Sigma_{U(i)}\left(\Upsilon(i)\right)^T\right) \tag{16}$$

Since only the standard deviations of variables, the square roots of the diagonal entries in a covariance matrix, are of interest for the quality of the product, the standard deviations of the elements in $X'(k), X(k)$ and $\Delta X(k)$ can be extracted from their covariance matrices and written as vectors.

$$\hat{S}_X = \left[\sqrt{(\Sigma_X)_{11}} \quad \sqrt{(\Sigma_X)_{22}} \quad \cdots \quad \sqrt{(\Sigma_X)_{mm}} \quad \cdots\right]$$

$$\hat{S}_{X'} = \left[\sqrt{(\Sigma_{X'})_{11}} \quad \sqrt{(\Sigma_{X'})_{22}} \quad \cdots \quad \sqrt{(\Sigma_{X'})_{mm}} \quad \cdots\right]$$

$$\hat{S}_{\Delta X} = \left[\sqrt{(\Delta\Sigma_X)_{11}} \quad \sqrt{(\Delta\Sigma_X)_{22}} \quad \cdots \quad \sqrt{(\Delta\Sigma_X)_{mm}} \quad \cdots \right]$$

where $\hat{S}$ is a vector for standard deviations. $(\Sigma)_{ii}$ is the i^{th} diagonal entry of a matrix Σ. For instance, $(\Sigma_X)_{22}$ is the second diagonal entry in the covariance matrix for the state vector X without considering uncertainty.

Based on the definition of uncertainty shown as Eq. (7), the output uncertainty can be obtained as follows:

$$\zeta_{\Sigma}^{(k)} = \frac{\left\|\hat{S}_{\Delta X}\right\|_2}{\left\|\hat{S}_X\right\|_2} = \frac{\left\|\left[\sqrt{(\Delta\Sigma_X)_{11}} \quad \sqrt{(\Delta\Sigma_X)_{22}} \quad \cdots \quad \sqrt{(\Delta\Sigma_X)_{mm}} \quad \cdots \right]\right\|_2}{\left\|\left[\sqrt{(\Sigma_X)_{11}} \quad \sqrt{(\Sigma_X)_{22}} \quad \cdots \quad \sqrt{(\Sigma_X)_{mm}} \quad \cdots \right]\right\|_2}$$
$$= \frac{\sqrt{Tr\left(\left((k+1)\zeta\right)^2 \Upsilon(0)\Sigma_{X(0)}\left(\Upsilon(0)\right)^T + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)^2 \Upsilon(i)\Sigma_{U(i)}\left(\Upsilon(i)\right)^T\right)\right)}}{\sqrt{Tr\left(\gamma(0)\Sigma_{X(0)}\gamma^T(0) + \sum_{i=1}^{k}\left(\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right)\right)}} \quad (17)$$

where $\zeta_{\Sigma}^{(k)}$ is the uncertainty associated with the points in the state vector at the k^{th} station. $Tr(\cdot)$ is the operator to obtain the trace of a matrix. ζ measures the parameter and input uncertainty of the station model. Equation (17), called uncertainty model, describes the relationship among the uncertainty associated with the output state vector at the k^{th} station, the station number k and the uncertainty for the parameters and inputs of the station model.

Based on the uncertainty model shown as Eq. (17), the uncertainty models for several special and interesting cases will be derived and their uncertainty propagation characteristics will be analyzed in the following sections.

3.3.3.1 No fixture variation and no uncertainty associated with the variation.

Assuming that there is no fixture variation or the fixture variation is negligible compared with the incoming part variation, the term considering the uncertainty associated with the fixture variation in Eq. (17) can be neglected. Therefore, the equation is rewritten as follows:

$$\zeta_{\Sigma}^{(k)} = \frac{\sqrt{Tr\left(\left((k+1)\zeta\right)^2 \Upsilon(0)\Sigma_{X(0)}\left(\Upsilon(0)\right)^T\right)}}{\sqrt{Tr\left(\gamma(0)\Sigma_{X_0}\gamma^T(0)\right)}} = \left((k+1)\zeta\right) \tag{18}$$

where $\zeta_{\Sigma}^{(k)}$ is the accumulated uncertainty to the output state vector at the k^{th} station. From Eq. (18), it can be concluded that the output uncertainty increases as the model runs along the stations, which is also shown in Figure 3-3.

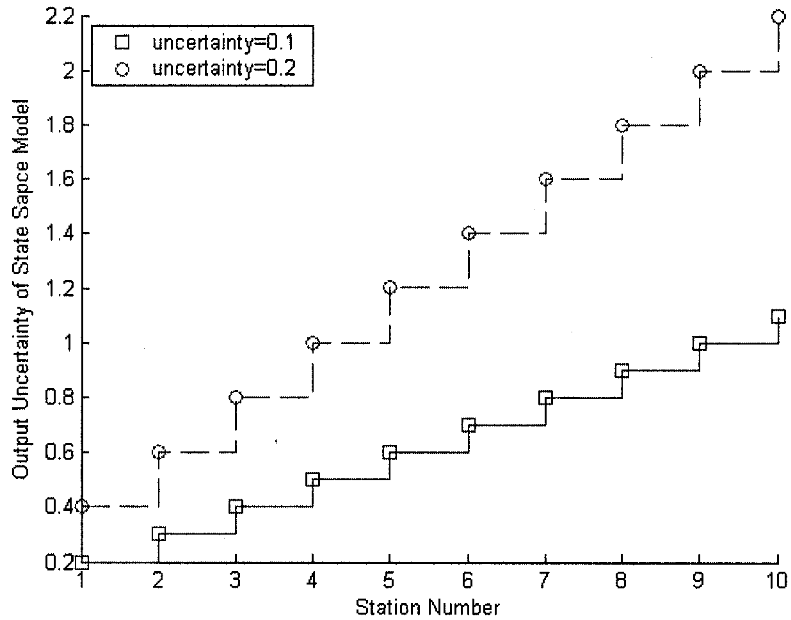

Figure 3-3. Parameter and incoming part uncertainty propagation

Figure 3-3 shows how the uncertainty propagates along the stations with the different parameter and input uncertainty (ζ). As shown in the figure, the uncertainty associated with the outputs of the state space model increases along the stations. For example, at the last station (the 10^{th} station) the uncertainty associated with the state vector is about 1.1 while the uncertainty is about 0.6 at the fifth station when the parameter and input uncertainties are equal to 0.1.

In addition, Figure 3-3 shows that the output uncertainty increases quicker as the parameter and input uncertainties become larger. For example, when the parameter and input uncertainties are equal to 0.1, the output uncertainty of the state space model is about 0.6 at the 5^{th} station. In comparison, the output uncertainty of the state space model is about 1.2 at the 5^{th} station when the parameter and input uncertainties are equal to 0.2.

Aware of the relationship between the uncertainties associated with the inputs and the parameters of the station model and the output uncertainty of the system model, the relationship among the output uncertainties of the station model and the system model will be derived and demonstrated because estimating the output uncertainty of the station model is easier than estimating the input and parameter uncertainties in reality.

From Eq. (18), the output uncertainty of the station model can be obtained by setting $k=1$ as follows:

$$\zeta_{\Sigma}^{(1)} = 2\zeta \tag{19}$$

where $\zeta_{\Sigma}^{(1)}$ is the output uncertainty at the station level and ζ is the parameter and input uncertainty for the station model.

Substituting the above equation into Eq. (18) gives the following equation

$$\zeta_{\Sigma}^{(k)} = \frac{(k+1)}{2}\zeta_{\Sigma}^{(1)} \quad (k=1,2,\cdots) \tag{20}$$

where $\zeta_{\Sigma}^{(k)}$ is the accumulated output uncertainty after running the state space model for k stations.

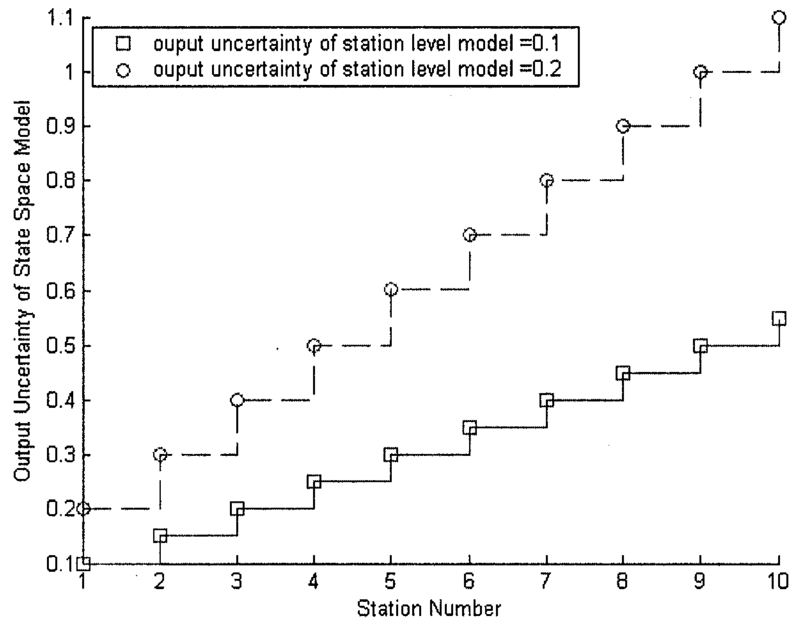

Figure 3-4. Output uncertainties of system and station models

Figure 3-4 demonstrates how the output uncertainty of the station model propagates along the stations when it is equal to 0.1 and 0.2, respectively. From this figure, the same conclusions as those from Figure 3-3 can be made.

3.3.3.2 No parameter uncertainty. In this section, it is assumed that there is only input uncertainty associated with incoming parts $X(0)$ and fixtures $U(i)$ $(i = 1, 2, \cdots, n)$ and no parameter uncertainty is associated with A and B in the state space model. Along with the assumption made for the uncertainty derivation before, the state space model is perfect or accurate enough in terms of its model parameters and model structure.

Equation (16) can be rewritten as follows:

$$
\begin{aligned}
\Delta\Sigma_{X(k)} &= \zeta^2\gamma(0)\Sigma_{X(0)}\gamma^T(0) + \sum_{i=1}^{k}\left(\zeta^2\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right) \\
&= \zeta^2\left(\gamma(0)\Sigma_{X(0)}\gamma^T(0) + \sum_{i=1}^{k}\left(\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right)\right) \\
&= \zeta^2\,\Sigma_{X(k)}
\end{aligned} \tag{21}
$$

Based on this equation and Eq. (17), the uncertainty associated with the state vector at the k^{th} station can be obtained as follows:

$$
\zeta_{\Sigma}^{(k)} = \frac{\left\|\hat{S}_{\Delta X}\right\|_2}{\left\|\hat{S}_X\right\|_2} = \frac{\sqrt{Tr\left(\Delta\Sigma_X\right)}}{\sqrt{Tr\left(\Sigma_X\right)}} = \zeta \tag{22}
$$

Eq. (22) shows that the output uncertainty of the state space model $\zeta_{\Sigma}^{(k)}$ is independent of k, the station number having run by the model. In the other words, the input uncertainty does not accumulate with running the model along the stations. Even though this conclusion is made under the assumption that the uncertainties for the fixtures $U(i)\left(i=1,2,\cdots,n\right)$ and the incoming parts $X(0)$ are the same, a similar conclusion can also be made if the uncertainties for $U(i)(i=1,2,\cdots,n)$ and $X(0)$ are not the same where a constant range instead of a constant numerical value is obtained for the output uncertainty of the state space model at different stations.

3.4 Case study

In this section, a case study is presented to illustrate the application of the proposed methodology in analyzing and monitoring the uncertainty of a state space model for a multi-stage system.

First, a procedure is proposed to evaluate the parameter uncertainty associated with the station model. Afterwards, an example is used to demonstrate how the parameter and input uncertainty of the station model propagates along the stations in a state space model.

In order to clearly explain this procedure, the model under study is assumed as follows even though it has not to be that:

$$y = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n$$

where y is the output of the model. $a_i \left(i = 1, 2, \cdots, n\right)$ are the model parameters. $x_i \left(i = 1, 2, \cdots, n\right)$ are the input variables of the model. The following procedure is used to evaluate the uncertainty associated with $a_i \left(i = 1, 2, \cdots, n\right)$.

1) Collect field data, $\left(y^m, X^m\right)$,

   where,

   y^m is the response of the m^{th} observation and it can be a vector though a scalar is used for the illustration in this paper.

   $X^m = \left\{x_1^m, x_2^m, \cdots, x_n^m\right\}$ is the m^{th} observation of the input variables $x_i \left(i = 1, 2, \cdots, n\right)$.

   m is the index for the observation in the collection of the field data

2) Obtain the optimal parameter set $\hat{a}_i \left(i = 1, 2, \cdots, n\right)$ for best fitting of $y = \hat{a}_1 x_1 + \hat{a}_2 x_2 + \cdots + \hat{a}_n x_n$ to the collected data set $\left(y^k, X^k, k = 1, 2 \cdots\right)$ using the least sum square method, and

3) Calculate the uncertainty for each parameter as follows using the quantification of uncertainty shown as Eq. (7):

$$\zeta_i = \sqrt{\frac{\left(\hat{a}_i - a_i\right)^2}{a_i^2}} = \frac{\left|\hat{a}_i - a_i\right|}{\left|a_i\right|} \quad \left(i = 1, 2, \cdots, n\right)$$

The procedure is used to estimate the parameter uncertainty for the models in this paper or the similar model even though the behind idea could be extended.

Aware of how to estimate the parameter uncertainty, the input uncertainties associated with $X(0)$ and $U(k)$ $\left(k = 1, 2, \cdots\right)$ in this paper can be obtained by analyzing the accuracy of the measurement systems used for $X(0)$ and $U(k)$ $\left(k = 1, 2, \cdots\right)$.

After calculating the parameter uncertainty and the input uncertainty, a case study will be conducted to illustrate how these uncertainties affect the accuracy of the final simulation result of a state space model for a multi-stage manufacturing system.

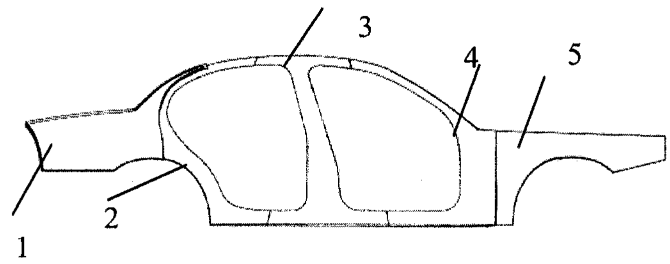

Figure 3-5. Side frame structure

The example used in this case study is shown in Figure 3-5. It is a side frame structure consisting of 5 components. Component 1 is the quarter panel, component 2 is the C pillar, component 3 is the B pillar, component 4 is the A pillar and component 5 is the front fender. All the components are assumed rigid.

The side frame structure is assembled in four stations and measured in the fifth station. In this first station, component 1 and component 2 are welded together. Component 3 is added in the second station. Other components are similarly added to the product one by one at each station until component 5 is added in the fourth station. In this example, only the in-plane variation is considered. The simplified components and their fixture schemes regarding this assembly process are shown in Figure 3-6. Each component has two fixtures. For instance, $\{P1, P2\}$ are the fixtures for Component 1 and $\{P3, P4\}$ are the fixtures for Component 2.

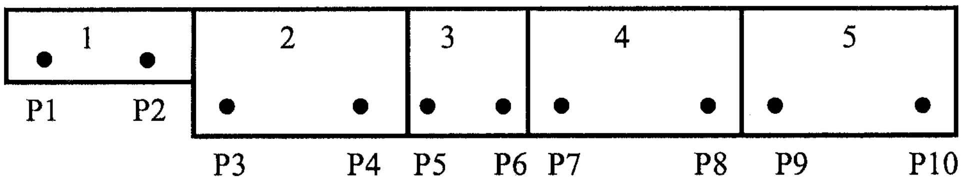

Figure 3-6. Simplified model and fixture schemes for the structure

Based on the geometry of the components and the fixture schemes shown in Figure 3-6, a state space model can be established for analyzing the variation propagation in the process for this product. The uncertainty analysis for this model will be conducted in the following several cases.

3.4.1 Case 1: uncertainties associated with all the parameters and inputs

In this case, the station model has the uncertainties associated with the state matrix A, input matrix B, incoming parts $X(0)$ and the fixtures $U(i)$ $(i = 1, 2, \cdots, n)$. The effects of these uncertainties on the output of the state space model will be illustrated.

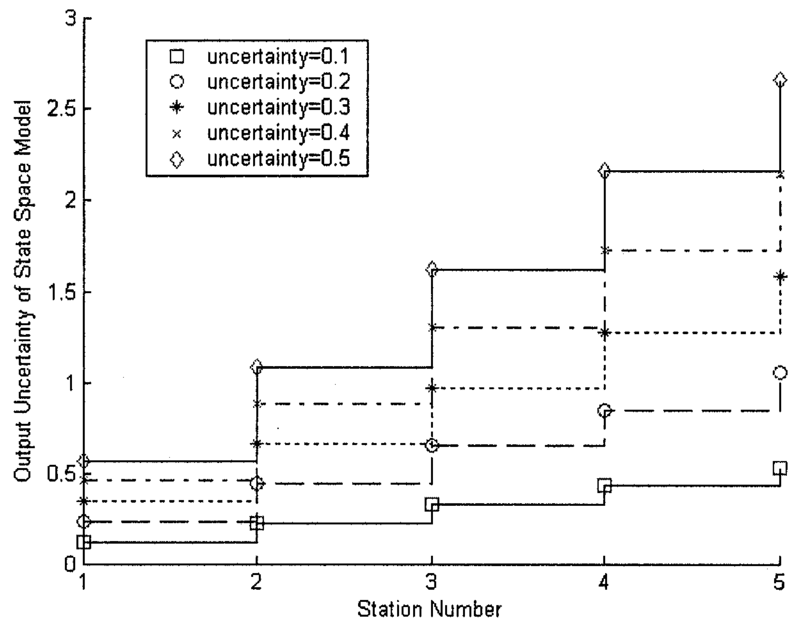

Figure 3-7. Input and parameter uncertainty propagation

In Figure 3-7, Y axis is the uncertainty for the output state vectors of each station. X axis is the number of stations having run by the model. Each curve represents the uncertainty propagation along the assembly stations for each case. Different curves represent the different levels for the input and parameter uncertainties. In this figure, it can be observed that the input and parameter uncertainties propagate and accumulate along the stations for all the uncertainty levels. In addition, it can also be noticed that the output uncertainty is larger at the same station when the input and parameter uncertainty is larger. For example, at the second station, the output uncertainty is about 0.2 when the input and parameter uncertainty is 0.1. In comparison, the output uncertainty is about 1.1 when the input and parameter uncertainty is 0.5.

3.4.2 Case 2: input uncertainty only

In this case, the station model has only the uncertainties associated with the incoming parts $X(0)$ and the fixtures $U(i)$ $(i=1,2,\cdots,n)$ in all the stations. There are no uncertainties associated with the state matrix and the input matrix. The propagation and accumulation of these uncertainties on the simulation results is illustrated.

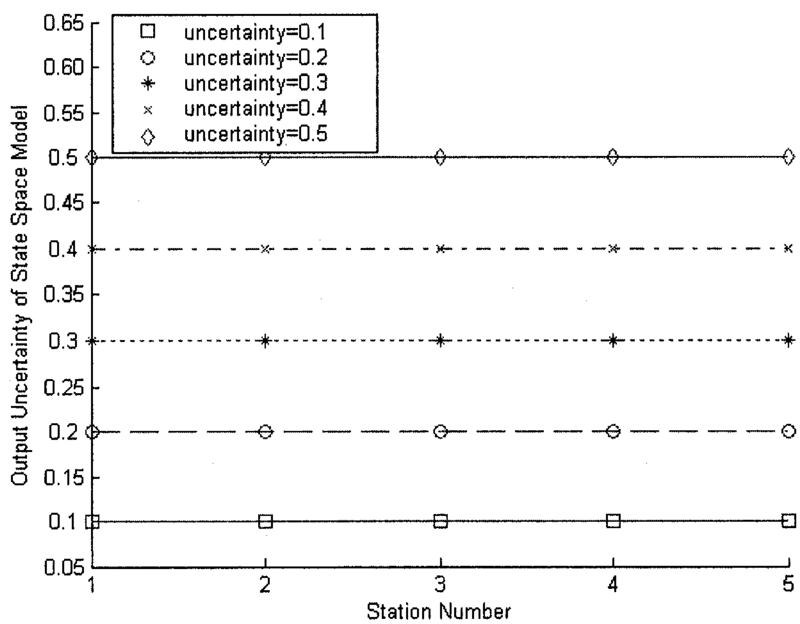

Figure 3-8. Input uncertainty propagation

X axis and Y axis have the same definitions as those in Figure 3-7. As shown in the figure, all the lines are horizontal and therefore the input uncertainties do not propagate and accumulate along the stations. This characteristic was explained in Section 3.3.3.2. For example, the uncertainty for the output state vectors at all the stations is equal to 0.2 when the input uncertainty is equal to 0.2. However, it can also be noticed that the output uncertainty is larger when the input uncertainty to the model is larger. For instance, the output uncertainty is about 0.1 when the input uncertainty is 0.1 while the output uncertainty is about 0.4 when the input uncertainty is about 0.4.

3.4.3 Case 3: uncertainties only associated with the parameters

In this case, the station model has the only uncertainties associated with the state matrix A and the input matrix B. There are no uncertainties associated with the incoming pars and the fixtures in all the stations. The effects of the state and input matrix uncertainties on the output of the state space model are illustrated.

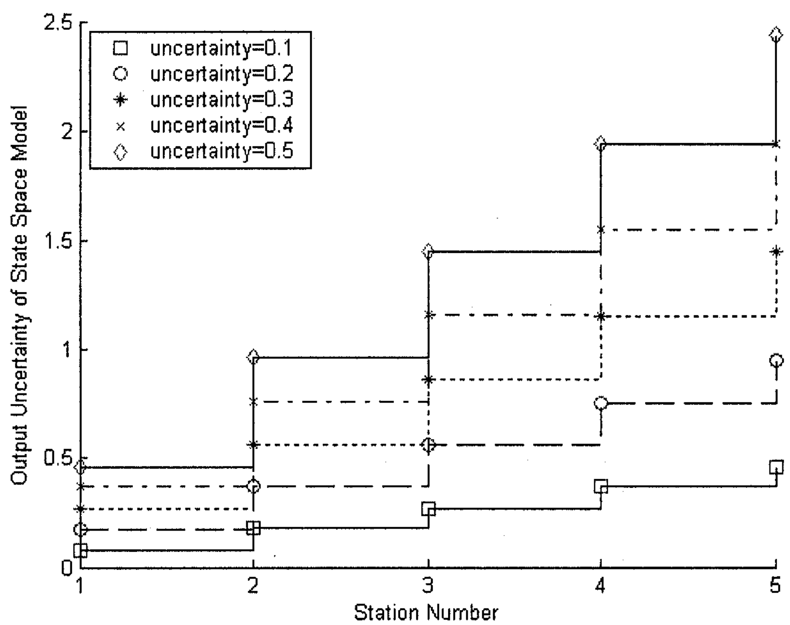

Figure 3-9. Parameter uncertainty propagation

In Figure 3-9, X axis and Y axis have the same definitions as those in the last two cases. As before, the same conclusion can be made that the output uncertainty is larger when the state matrix and input matrix uncertainties are larger. In addition, the state matrix and input matrix uncertainties propagate and accumulate with running the state space model along the stations.

Compared with the results in the Case 1 where the state space model has the uncertainty associated with the state matrix, input matrix, incoming parts and fixtures, the output uncertainty in this case is smaller for the same level of station level uncertainty. For instance, the output uncertainty is about 1.3 in the fourth station when the model has uncertainties with the state matrix, input matrix, incoming parts and fixtures at the level of 0.3. In comparison, the output uncertainty is about 1.1 in the third station when the model has only uncertainties associated with the state matrix and the input matrix at the same level of 0.3. The reason is that the uncertainty sources in the case 1 are more than those in the case 3.

3.5 Applications of the uncertainty model

The uncertainty model shown as Eq. (17) may have several applications such as uncertainty estimation and propagation analysis, simulation model verification and validation, risk analysis and management, model calibration and others. This section demonstrates these applications by using the uncertainty model in the calibration of variation simulation models in multi-stage manufacturing systems.

As mentioned before, one concern in the application of the variation simulation models for multi-stage manufacturing systems is uncertainty propagation and accumulation. When the uncertainty associated with the output of a state space model at some station is too large, calibrations have to be conducted to the state space model. For example, as explained in Section 3.1 and Section 3.3, the simulation output of the $(k-1)^{th}$ station model in a state space model, *X'(k),* is usually inputted into k^{th} station model of the state space model. However, if the uncertainty associated with *X'(k)* is too large, the measurement of the output at the $(k-1)^{th}$ station in the physical system instead of the simulation output *X'(k)* is inputted to the k^{th} station model. In the applications of the variation simulation models for multi-stage manufacturing systems, decisions about when the state space model has to stop running for a calibration is made based on the experience without a methodology to analyze and quantify the propagated uncertainty. This section illustrates how to use the proposed uncertainty model to estimate the maximal number of stations which a state space model can run before the calibration is required.

Equation (17) describes the relationship among the accumulated uncertainty to the output of a state space model, the input and the parameter uncertainties, and number of stations running by the state space model. It can be seen that the station number can be obtained from Eq. (17) with the predesignated target for the output uncertainty of the state space model and input and parameter uncertainties of the station model known, the number can be regarded as the maximal number of stations which the model can run to achieve the simulation results with some level of accuracy. Since Eq. (17) generally can only be solved for the maximal number of stations using numerical methods, the bounds of the output uncertainty at the k^{th} station given by the following two equations may be beneficial to the estimation of the maximal number of station.

$$\zeta_{\Sigma}^{(k)} \leq \frac{((k+1)\zeta)\sqrt{Tr\left(\Upsilon(0)\Sigma_{X(0)}(\Upsilon(0))^T+\sum_{i=1}^{k}\left(\Upsilon(i)\Sigma_{U(i)}(\Upsilon(i))^T\right)\right)}}{\sqrt{Tr\left(\gamma(0)\Sigma_{X_0}\gamma^T(0)+\sum_{i=1}^{k}\left(\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right)\right)}} \\ =((k+1)\zeta)=\left(\frac{(k+1)}{2}\zeta_{\Sigma}^{(1)}\right) \tag{23}$$

$$\zeta_{\Sigma}^{(k)} \geq \frac{(2\zeta)\sqrt{Tr\left(\Upsilon(0)\Sigma_{X(0)}(\Upsilon(0))^T+\sum_{i=1}^{k}\left(\Upsilon(i)\Sigma_{U(i)}(\Upsilon(i))^T\right)\right)}}{\sqrt{Tr\left(\gamma(0)\Sigma_{X_0}\gamma^T(0)+\sum_{i=1}^{k}\left(\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right)\right)}} \\ =2\zeta=\zeta_{\Sigma}^{(1)} \tag{24}$$

where $\zeta_{\Sigma}^{(k)}$ is the uncertainty associated with the output of a state space model at the k^{th} station; $\zeta_{\Sigma}^{(1)}$ is the output uncertainty of the station model which is defined in Section 3.3.3.1; ζ is input and parameter uncertainty for station models.

Before the discussion about these two equations, it has to be pointed out that the derivations of both Eq. (23) and Eq. (24) and therefore the conclusions from these two equations are based on the assumption that station models have uncertainties associated with all the model parameters and the inputs. Equation (23) provides the upper bound of the output uncertainty at the k^{th} station. On the other hand, Eq. (24) provides the lower bound equal to the output uncertainty of station models. It also can be implied from Eq. (24) that the predesignated target for the output uncertainty of the state space model less than the output uncertainty of station models cannot be achieved no matter how often the model is calibrated. This implication is important in evaluating the applicability of a model.

Based on the uncertainty model shown as Eq. (17), the maximal number of stations for model calibration will be solved in a close form for the following two situations.

3.5.1 No fixture variation and no uncertainty associated with the variation

As same as the situation in Section 3.3.3.1, it is assumed that no fixture variation is considered in the variation simulation model and therefore no uncertainty associated with the fixture variation in Eq. (17).

Based on Eq. (20), the following equation can be derived to describe the relationship between the output uncertainty of the station model and the maximal station number that the state space model can run before the calibration is needed to achieve the simulation results with some accuracy.

$$K = floor\left(2\frac{\zeta_{\Sigma}}{\zeta_{\Sigma}^{(1)}} - 1\right) \tag{25}$$

where $floor()$ is the operator to round a value down to the nearest integer less than or equal to the value; K is the maximal number of stations which the model can run before a calibration is required to keep the output uncertainty of the state space model less than the predesignated target ζ_{Σ} if the output uncertainty of the station model is equal to $\zeta_{\Sigma}^{(1)}$.

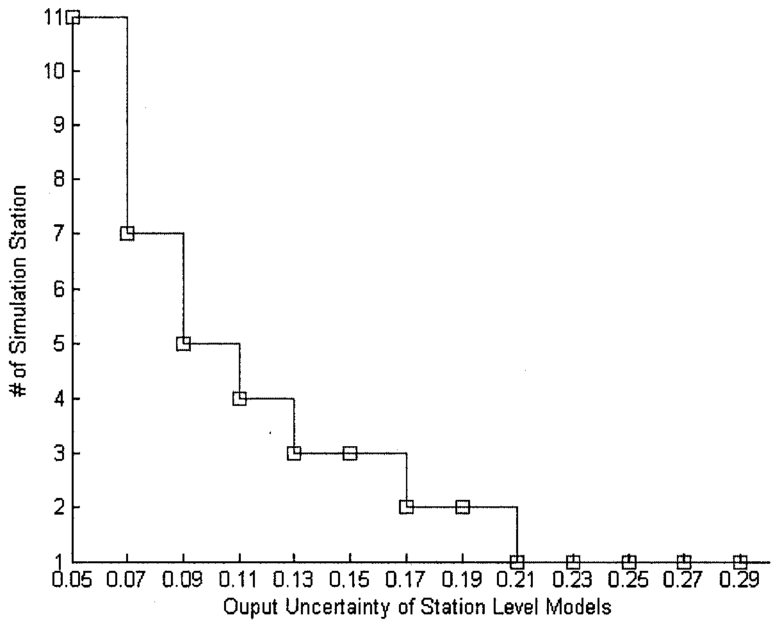

Figure 3-10. Number of simulation station before calibration vs. uncertainty of station model

In the example used in Figure 3-10, the target value ζ_Σ for the output uncertainty of the state space model is 0.3. This figure shows the maximal number of stations before the calibration is needed to the state space model in order to keep the output uncertainty less than 0.3. For example, if the output uncertainty of the station model is 0.05, the state space model has to be calibrated every 11 stations to guarantee the output uncertainty is less than 0.3. Moreover, it is of interest to notice that the station model has to be calibrated every station to keep the output uncertainty of the state space model less than 0.3 when the output uncertainty of the station model is greater than 0.21. All these observations are obtained under an assumption that the state space model is as good as the station model after calibration.

3.5.2 No parameter uncertainty

This situation is the same as that in Section 3.3.3.2 where only input uncertainty is associated with incoming parts and fixtures.

From the discussion in Section 3.3.3.2, it is known that the output uncertainty of a state space model is only depending on the output uncertainty of station models. Therefore, the calibration for the state space model is not necessary. The capability of a state space model for a multi-stage system only depends on the predesignated uncertainty target for the state space model and the uncertainty of station models.

3.6 Summary and conclusions

In this paper, the characteristics and the sources of the uncertainty for the simulation models of a multi-stage manufacturing system were analyzed. A general uncertainty model based on a state space model was established. Guidelines for model calibration were also established using the uncertainty model. In addition, a case study based on this uncertainty model was conducted. The following conclusions can be drawn:

1) In general, the uncertainties will propagate and accumulate to the simulation results while running a state space model along the stations in a multi-stage system. Therefore, the accuracy and fidelity of the simulation results is reduced.

2) The output uncertainty of a station model which is related to the uncertainties associated with the state matrix, the input matrix, the state

vector and the input vector plays a critical role in the output uncertainty of the state space model.

3) The input uncertainty, which includes the incoming part uncertainty and the fixture uncertainty, dose not propagate or accumulate to the output with running the state space model along the stations of a system even though it does induce some uncertainties on the simulation results of station models.

Although the conclusions were made based on the variation simulation models for multi-stage assembly systems with some assumptions, they could be extended to the similar models in the other fields along with the proposed methodologies.

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CHAPTER 4

TOLERANCE ALLOCATION CONSIDERING UNCERTAINTY IN VARIATION SIMULATION MODELS

Abstract

Tolerance allocation is an indispensable tool in product and process developments. In the optimization for tolerance allocation, variation simulation models play essential roles in describing the tolerance relationship between a product and its components. Since uncertainty is an inevitable characteristic of simulation models, it is important to understand the impacts of the uncertainty on tolerance allocation. This paper proposes a formulation for tolerance allocation considering uncertainty based on a Reliability Based Design Optimization (RBDO) method. A case study is conducted to illustrate the implementation of the formulation. The results of the case study demonstrate the uncertainty impacts on the allocated tolerances for the components of a product.

4.1 Introduction

Tolerance allocation and tolerance analysis are two indispensable tools in product development cycles and process design phases. Tolerance allocation, also referred to as tolerance synthesis, is a procedure to determine the tolerances of the components for a product and/or a process using appropriate rules to satisfy the tolerance targets of the product that are known from design requirements [Chase and Greenwood,1988]. Tolerance analysis, on the other hand, is to calculate the tolerances for a final product by stacking up the tolerances of its components and/or manufacturing tools. As shown in Figure 4-1, tolerance allocation is performed in the early phase of a product development cycle, before any parts have been produced and tooling ordered [Lee and Woo, 1990; Chase and Parkinson, 1991], and tolerance analysis is performed as an inner loop in tolerance allocation.

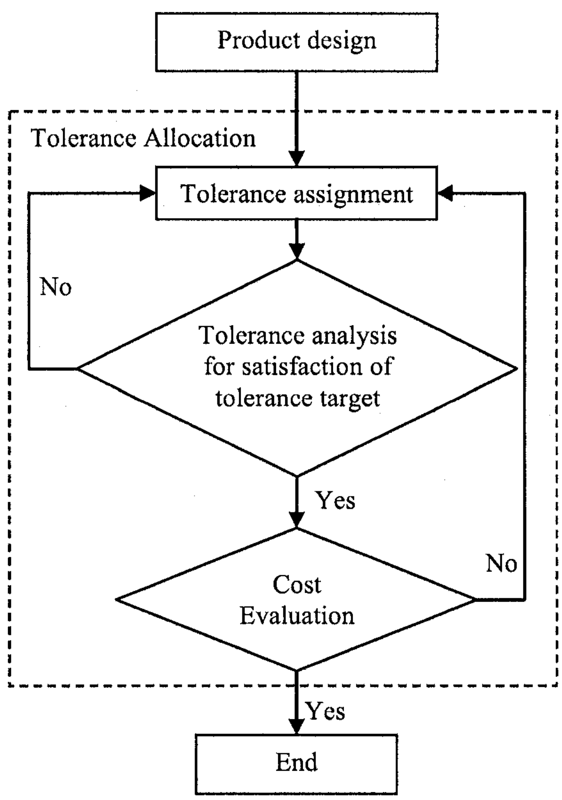

Figure 4-1. Tolerance analysis vs. tolerance allocation

Many studies have been conducted for tolerance allocation [Speckhart, 1972; Lee and Woo, 1990; Chase et al., 1990; Choi et al., 2000; Ding et al., 2000; Hong and Chang, 2002; Shiu et al., 2003; Zhong et al., 2002; Li et al., 2004]. In their research, tolerance allocation was generally formulated as a constrained optimization problem shown in the following by treating cost minimization as the objective function and requirements for product tolerances as constraints:

$$\min f_c(T_1, T_2 \cdots T_i) \tag{1a}$$

subject to

$$g_1(T_1, T_2 \cdots T_i) \le \hat{T}_A \tag{1b}$$

$$\begin{aligned} g_2(T_1, T_2 \cdots T_i) &\le 0 \\ h(T_1, T_2 \cdots T_i) &= 0 \end{aligned} \tag{1c}$$

where f_c represents the cost function, T_i $(i = 1, 2 \cdots)$ are optimization variables and their results are the allocated tolerances for components; Eq. (1b) represents the constraint that the resultant tolerance for a final product consisting of the components with the allocated tolerances must be less than the pre-designated tolerance target; $\hat{T}_A$ represents the pre-designated tolerance target which generally is the dimensional quality specifications for final products. Since tolerance is the allowed level of variation, variation simulations are always utilized in Eq. (1b) to calculate the final product tolerance from the tolerances of its components and manufacturing tools. Equation (1c) in the formulation may include the constraint related to the process capability of a system and other mathematical constraints for optimization.

Current research about tolerance allocation has been focusing on choosing cost functions and /or variation simulation models for different applications and accordingly using different optimization methods such as integer programming, linear programming, sequential linear programming, sequential quadratic programming, Lagrange multipliers and others to solve the problem. However, none of the research considered the uncertainties associated with the involved simulation models in their formulations. Uncertainty is the inaccurate and inevitable characteristic of a model compared with its simulated physical systems. It was asserted that it is impossible to specify, accurately and simultaneously, the values of the physical variables that describe the behavior of a physical system in Heisenberg Uncertainty Principle (HUP). Therefore, it is necessary to consider the uncertainties associated with the simulation models in tolerance allocation. This paper develops a tolerance method, proposes an optimization formulation, and performs a case study including uncertainty in tolerance allocation to demonstrate the uncertainty impacts on allocation results.

This paper is organized as follows. Section 4.2 reviews variation simulation models and their uncertainty. Section 4.3 proposes an optimization formulation based on a Reliability-Based Design Optimization (RBDO) method. In Section 4.4, a case study of tolerance allocation for a compliant assembly is conducted to illustrate the uncertainty effects on the allocation results. A discussion about the case study results is also performed. Section 4.5 summarizes and concludes this paper.

4.2 Variation simulation models and their uncertainties

4.2.1 Variation simulation models

As mentioned in Section 4.1, variation simulation models, representing the variation relationship between a final product and its components and process, are typically employed in the constraint shown as Eq.(1b). The most commonly used variation simulation models include Worst Case model and Root Sum Square (RSS) models. A detailed review about these two models was given by Chase and Parkinson [1991] and Juster [1992]. However, both Worst Case and RSS models are difficult to be applied to the tolerance allocation for complex two or three dimensional assemblies. Although Monte-Carlo Simulation (MCS) method can be applied to the tolerance allocation for complex assemblies [Doydum and Perreira, 1992; Lin et al., 1997], the allocation implementation is time consuming and computationally intensive since MCS is a sample-based method. After Chase and Greenwood [1988] stated that inclusion of realistic physical/functional models of integrated product and manufacturing processes is especially important for the current technology of manufacturing complex products, more and more variation simulation models for manufacturing systems were developed and utilized in tolerance allocation.

Since manufacturing processes are typically multilevel and hierarchical systems, the variation propagation and accumulation in the systems is usually modeled by the state space form shown in Eq.(2) using the different definitions for the state matrix $A(k)$, the state vector $X(k)$, the input matrix $B(k)$, and the input vector $U(k)$

$(k = 1, 2, \cdots, N)$ [Mantripragada and Whitney, 1999; Jin and Shi, 1999; Camelio et al., 2003].

$$X(k) = A(k)X(k-1) + B(k)U(k) \tag{2}$$

In general, the model can be integrated and written as follows:

$$X(k) = \Phi(k,1)X(0) + \sum_{j=1}^{k} \left(\Psi(k,j)U(j) \right) \tag{3}$$

where,

$$\begin{aligned} &\Phi(k,j) = A(k) * A(k-1) * \cdots * A(j+1) * A(j) \quad (k \ge j) \\ &\qquad \text{and} \quad \Phi(j,j) = A(j) \\ &\Psi(k,j) = A(k) * A(k-1) * \cdots * A(j+1) * B(j) \qquad (k \ge j) \\ &\qquad \text{and} \quad \Psi(j,j) = B(j) \end{aligned}$$

N is the station number in a manufacturing system; and $X(0)$ is the deviation vector for the source points, including Key Product Characteristics (KPC) and/or Key Control Characteristics (KCC) points for all the incoming parts. Neglecting noise and disturbance effects, Eq. (3) describes how the deviation of each part or subassembly propagates and accumulates into the final product during manufacturing processes. In order to obtain the equations for variance propagation, it is assumed that the different sources of variation are independent of each other. For instance, the incoming part variation is independent of the fixture variation. Under this assumption, the following equation about variances can be derived from Eq. (3):

$$
\begin{aligned}
\Sigma_{X(k)} &= \Phi(k,1)\Sigma_{X(0)}\Phi^T(k,1) + \sum_{j=1}^{k}\left(\Psi(k,j)\Sigma_{U(j)}\Psi^T(k,j)\right) \\
&= \gamma(0)\Sigma_{X(0)}\gamma^T(0) + \sum_{j=1}^{k}\left(\gamma(j)\Sigma_{U(j)}\gamma^T(j)\right)
\end{aligned}
\tag{4}
$$

where, $\gamma(j) = A(k)A(k-1)\cdots A(j+1)B(j) \quad (j = 0,1\cdots,k; \quad B(0) = I)$; $\Sigma_{X(k)}$ is the covariance matrix of the deviation of the points in the state vector $X(k)$; $\Sigma_{U(j)}$ is the covariance matrix of the errors for the fixtures at the j^{th} station; and $\Sigma_{X(0)}$ is the covariance matrix for the source points on all the incoming parts.

4.2.2 Uncertainties in variation simulation models

Zhao et al. [1995] defined uncertainty as the differences or errors between models and the reality. Oberkampf et al. [1999] described uncertainty as a potential deficiency in any phase or activity of a modeling process due to a lack of knowledge. Delaurentis and Mavris [2000] provided the definition of uncertainty as incompleteness in knowledge (either in information or context) which causes model-based predictions to differ from the reality in a manner described by some distribution functions.

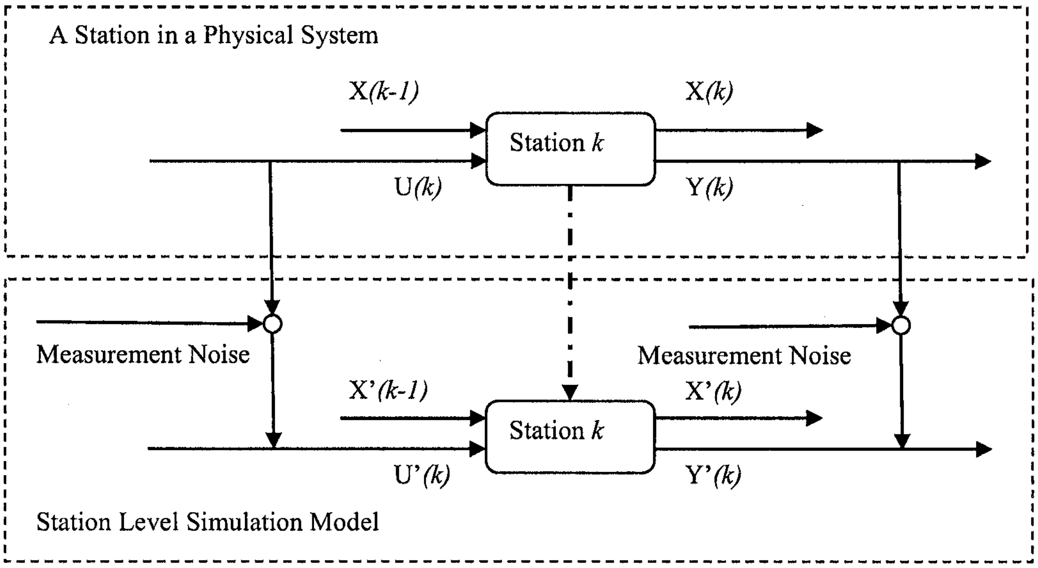

Figure 4-2. Uncertainty analysis for the station model

As illustrated in Figure 4-2, Yue et al. [2006] classified the uncertainty of the variation simulation models for multi-stage manufacturing systems as the input uncertainty (associated with $U'(k)$), the propagated uncertainty (associated with $X'(k)$ and $X'(k-1)$), the station model uncertainty (associated with the station model k) and the system mode uncertainty (associated with the system model consisting al the station models). The last two types of uncertainty are also referred to as model uncertainty which includes the model parameter uncertainty and the model structure uncertainty. The uncertainty sources in the variation simulation models were analyzed and summarized as measurement errors, assumptions and simplification, propagation, computation errors and other errors.

In order to quantify the uncertainty, a formula was proposed as follows:

$$\zeta = \frac{\|Y' - Y\|_2}{\|Y\|_2} = \frac{\|\Delta Y\|_2}{\|Y\|_2} \tag{5}$$

where ζ is the uncertainty associated with a variable. Y' and Y are the values with and without considering uncertainty of the under study variable which can be the input variable or the parameters of a model. Y' and Y can be scalars or vectors. ΔY is the difference in the values of the variable with and without considering uncertainty. $\| \ \|_2$ is the second norm operator.

It was concluded by Yue et al. [2006] that a model, considering uncertainties, cannot provide a prediction as one deterministic value but a range or a random value with a statistical distribution such as the uniform distribution. Therefore, Y', the prediction considering uncertainty, was represented as follows by assuming that Y' and Y have the same structure:

$$Y' \sim U\left(Y - \Delta Y \quad Y + \Delta Y\right) = U\left((1-\zeta)Y \quad (1+\zeta)Y\right) \tag{6}$$

In order to illustrate uncertainty propagation and accumulation while simulation models running along the stations of a system, an uncertainty model was developed from the state space model shown in Eq. (2) based on the definitions and the qualification of uncertainty. The uncertainty model is shown as follows:

$$\zeta_{\Sigma}^{(k)} = \frac{\sqrt{Tr\left(\left((k+1)\zeta\right)^2 \Upsilon(0)\Sigma_{X(0)}\left(\Upsilon(0)\right)^T + \sum_{i=1}^{k}\left(\left((k-i+2)\zeta\right)^2 \Upsilon(i)\Sigma_{U(i)}\left(\Upsilon(i)\right)^T\right)\right)}}{\sqrt{Tr\left(\gamma(0)\Sigma_{X(0)}\gamma^T(0) + \sum_{i=1}^{k}\left(\gamma(i)\Sigma_{U(i)}\gamma^T(i)\right)\right)}} \tag{7}$$

where $\zeta_{\Sigma}^{(k)}$ is the uncertainty associated with the standard deviations of the points in the state vector at the k^{th} station; $Tr(\cdot)$ is the operator to obtain the trace of a matrix; $\Sigma_{X(0)}$ and $\Sigma_{U(i)}$ are the covariance matrices for the incoming parts and the fixtures at the i^{th} station, respectively; ζ measures the parameter and input uncertainty of the station model; similar to the definition of $\gamma(j)$ in Eq.(4), $\gamma'(j)$ is defined as: $\gamma'(j) = A'(k)A'(k-1)\cdots A(j+1)B(j)$ $\left(j = 0,1\cdots,k;\ B(0) = I\right)$. The detailed derivation of this uncertainty model can be referred to the paper.

4.3 Proposed formulation

In order to incorporate uncertainty and/or variation into optimization, methodologies for robust design, Design for Six Sigma (DFSS) and probabilistic/ Reliability Based Design Optimization (RBDO) methods were developed. Robust design proposed by Dr. Genichi Taguchi in 1970s was initially used to improve quality and reliability in products. Several formulations of robust design exist for optimization under uncertainty [Parkinson et al, 1993; Parkinson, 1995; Parkinson, 1997; Mavris et al., 1999; Du and Chen, 2000; Gu et al., 2000; Kalsi et al., 2001; Jung and Lee, 2002; Hirokawa and Fujita, 2002]. In comparison, DFSS is used for high reliability products through optimization. In DFSS, the dispersion of a variable is accounted for by its standard deviation in both objective functions and constraints equations. Several formulations were also presented for the different applications [Fu et al., 2002; Antony and Coronado, 2002; Choudri, 2004; Sokovic et al., 2005]. In RBDO, all uncertain quantities are assumed random with known distributions. The formulations based on this method were described in several literatures [Siddall, 1983; Melchers, 1987; Ditlevsen and Madsen, 1996; Tu et al., 1999; Chan et al., 2005]. Although all the formulations dealing with uncertainty are based on the different methods, the differences among them sometimes are hardly identified.

In this paper, an optimization formulation is proposed using a RBDO method to incorporate the uncertainty associated with the outputs of variation simulation models. First, the assumptions for this formulation are clarified as follows:

(1) The uncertainty associated with the cost function f_c shown in Eq. (1a) is not considered, and

(2) The constraints formulated by Eq. (1c) are not included.

with these assumptions, the optimization formulation is proposed as follows:

$$\min f_c\left(T_1, T_2 \cdots T_i\right) \tag{8a}$$

subject to

$$\Pr\left(g_1^{'}\left(T_1, T_2 \cdots T_i\right) \leq \hat{T}_A\right) \geq \left(1 - P_r\right) \tag{8b}$$

where Eq. (8a), the same as Eq. (1a), is a cost function; $g_1^{'}$ is a variation simulation model considering uncertainty; $\Pr(\cdot)$ is the cumulative distribution function of the output of $g_1^{'}$; T_i $(i = 1, 2, \cdots)$ and $\hat{T}_A$ have the same definitions as in Eq. (1b); P_r is related to the design reliability by specifying the optimal design will have $\left(1 - P_r\right)$ reliability of satisfying the constraint considering the uncertainty.

4.3.1 Cost model

Cost model shown in Eq. (8a) and Eq. (1a) describes a tolerance-cost relationship in manufacturing systems. Many cost models regarding the tolerances of a part have been proposed for different tolerance allocation schemes [Wu et al., 1988; Chase et al., 1990]. Most of the models utilize the reciprocal, reciprocal squared and negative exponential functions to describe the negative correlation between the fabrication costs of a part and its tolerances. Since choosing different functions as the cost model dose not change the conclusions of this research, a negative exponential function shown in the following is chosen for this example:

$$f_c(T_1, T_2, \cdots, T_n) = \sum_{i=1}^{L}\left(w_i \times C_i(T)\right) = \sum_{i=1}^{L}\left(w_i \times \left(c_{i1} + c_{i2} \times e^{-c_{i3}T}\right)\right) \tag{9}$$

where $C_i(T)$ is the cost model for the i^{th} component of a product; L is the total number of components in a product; c_{i1} represents the fixed cost for the process of the i^{th} component fabrication; c_{i2} along with c_{i1} determines the upper limit of the cost to produce the i^{th} component. c_{i3} describes how the cost is sensitive to the changes of the tolerance of the i^{th} component; w_i is the weight coefficient for the cost of the i^{th} component.

4.3.2 Variation-tolerance relationship

In order to describe the tolerance relationship between a final product and its components in the unequal constraint shown as Eq. (8b) using a variation simulation model, a connection between tolerances and variations has to be established.

By assuming manufacturing systems are 6 sigma processes without mean shift, the connection can be obtained as:

$$\begin{aligned} T_i &= 3\sigma_i \\ \sigma_i &= \sqrt{\left(diag(\Sigma)_i\right)} \\ T_i &= 3\sqrt{\left(diag(\Sigma)_i\right)} \end{aligned} \tag{10}$$

where T_i and σ_i is the tolerance and the standard deviation for a KPC or KCC point; $diag()_i$ is the operator to obtain the i^{th} diagonal entry of a matrix; Σ represents a covariance matrix.

Substituting Eq. (10) into Eq. (4) gives an equation for the tolerance relationship between a final product and its components as follows:

$$g\left(T_1, T_2, \cdots, T_n\right) = T_A \tag{11}$$

where T_A is the tolerance for the final product when its components have the tolerances of T_i $\left(i = 1, 2, \cdots, n\right)$.

4.4 Case study

An example shown as in Figure 4-3, a vehicle side frame structure, is used in this case study to illustrate the proposed formulation. As shown in the figure, the structure consists of three parts: part 1, part 2 and part 3.

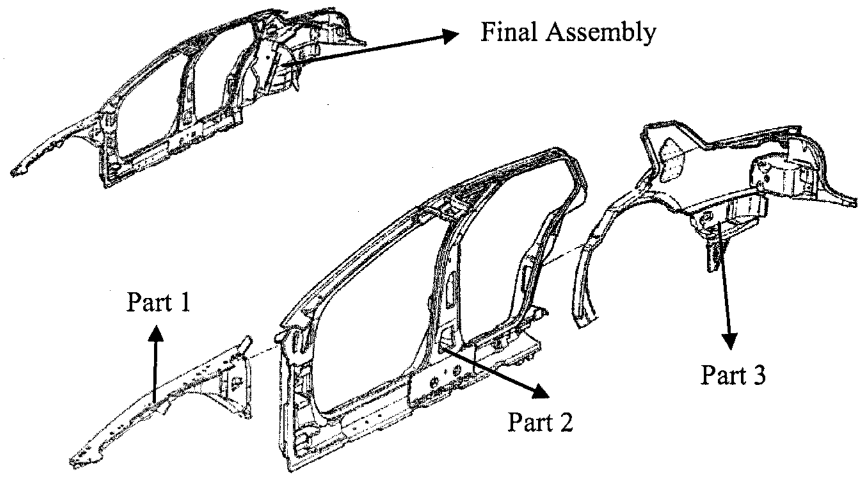

Figure 4-3. Side frame structure

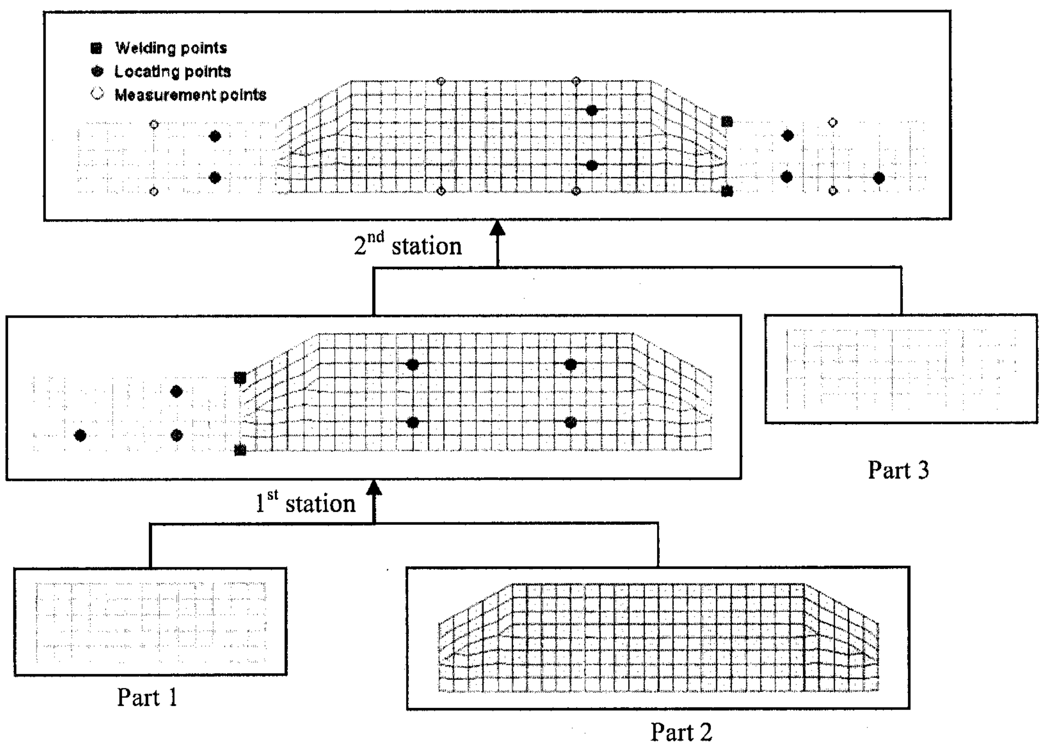

Figure 4-4. Assembly process and FEM models

In Figure 4-4, the assembly process for the side frame structure is illustrated and the simplified FEM models for the final assembly and its components are shown. The welding points, locating points and measurement points are also shown in the figure as solid squares, solid circles and circles, respectively.

As shown in the figure, the side frame structure is sequentially welded together at two stations. At the first station part 1 and part 2 are welded as a subassembly which is subsequently assembled with part 3 at the second station for the final product. There are two welding points between part 1 and part 2, and between the subassembly and part 3. Therefore, there are two welding points for part 1 and part 3 at one of their ends and four welding points for part 2 at both of its ends. As shown in the figure through the locating points, the fixture schemes for part 2 at the first station and the subassembly at the second station are "4-2-1" and those for part 1 and part 3 are "3-2-1". At the end of the assembly process, eight points are measured at the second station for monitoring the dimensional quality of the final product. The variation only in the out of plane direction is considered in this example. In addition, for this example, it is assumed the fixtures at both stations and locating points for all the parts are perfect. Therefore, the only variation source is the dimensional variation of the welding points on the parts. Therefore, using the proposed formulation, the tolerances will be optimally allocated to the welding points for each part with minimum cost to ensure the tolerance of the final product is within the pre-designated dimensional requirements.

In addition, in order to simulate the variation propagation for this structure, Finite Element Analysis has to be applied to obtain the state and input matrices. The simplified FEM models for the structure and its components are also shown in Figure 4-4. In these models, the length is equal to 0.8m for part 1 and part 3 and 1.4m for part 2. The thickness is equal to 2mm and the material is mild steel with Young's modulus $E = 3.06Gpa$ and Poisson's ratio $\nu = 0.3$ for all the parts.

4.4.1 Variation simulation model for compliant assembly systems

Since the side frame structure in this case study is considered as a compliant assembly, the variation simulation model for compliant assemblies will be employed in the tolerance allocation optimization.

In order to describe the dimensional relationship between a compliant assembly and its components at the station level, Liu et al. [1996] and Liu and Hu [1997] proposed a linear model:

$$V_w = S \cdot V_u \tag{12}$$

where V_w and V_u are vectors that represent the dimensional variation of the Key Product Characteristics (KPCs) of an assembly and its components, respectively; and S is the sensitivity matrix considering the deformations and springbacks of products in assembly processes. It can be obtained using the influence coefficient method for complex products.

Based on the station model shown as Eq. (12), Camelio et al. [2003] extended the state space model expressed in Eq. (2) to include part compliance. The extended model, shown as Eq. (13), describes the dimensional deviation propagation along the stations for a compliant assembly process.

$$X(k) = \left(S(k) - D(k) + I\right)\left(X(k-1) + M(k)\left(X(k-1) - U(k)_{3-2-1}\right)\right) \\ -\left(S(k) - D(k)\right)\left(U(k)_{n-3} + U(k)_g\right) + v(k) \tag{13}$$

where $S(k)$ is the sensitivity matrix which describes the induced (sub)assembly deviation due to a unit deviation of the incoming parts at the k^{th} station. I is an identity matrix. The re-locating matrix, $M(k)$, explains how the state vector changes due to the change of the locating scheme from the previous station to the current station. On the other hand, the deformation matrix, $D(k)$, concerns the initial shape of incoming parts or subassemblies. $U(k)_{3-2-1}$ was defined as the deviation of the "3-2-1" fixtures at the k^{th} station. $U(k)_{n-3}$ was defined as the deviation of the "$n-2-1(n>3)$" fixtures at the k^{th} station. $U(k)_g$ was defined as the deviation of the assembly tools at the k^{th} station.

With an assumption that the fixtures at all the stations and the locating points for all the parts are prefect, Eq. (13) can be simplified as follows:

$$\begin{aligned} X(k) &= \left(S(k) - D(k) + I\right)\left(X(k-1) + M(k)X(k-1)\right) + v(k) \\ &= \left(S(k) - D(k) + I\right)\left(I + M(k)\right)X(k-1) + v(k) \\ &= A(k)X(k-1) + v(k) \end{aligned} \tag{14}$$

Equation (14) describes the deviation relationship between the final product and its components. Their variance relationship can be derived based on Eq. (14) as:

$$\Sigma_{X(k)} = \gamma(0)\Sigma_{X(0)}\gamma^T(0) \tag{15}$$

where $X(k)$ represents state vectors and $X(0)$ represents the deviations of the welding points for part 1, part 2 and part 3; k is station number which can be 1 or 2 in this example; $\Sigma_{X(k)}$ and $\Sigma_{X(0)}$ are the corresponding covariance matrices for the points in $X(k)$ and $X(0)$; $\gamma(0) = A(k)A(k-1)\cdots A(1)$.

Based on Eq. (15) and the variation-tolerance relationship explained in Section 4.3.2, the following equation can be obtained and used in expressing the unequal constraint in the formulations.

$$\begin{aligned} g\left(T_1, T_2, \cdots, T_n\right) &= \frac{1}{3}\max\left(diag\left(\Sigma_{X(k)}\right)\right) \\ &= \frac{1}{3}\max\left(diag\left(\gamma(0)\Sigma_{X(0)}\gamma^T(0)\right)\right) \end{aligned} \tag{16}$$

From Eq.(16), it can be seen that the largest tolerance among the measurement points is chosen to control the dimensional quality of the final assembly.

4.4.2 Formulation

In the formulations of this example, $\hat{T}_A$, the tolerance specification for the final product, is set as 2mm. In the other words, the maximal tolerance among all the measurement points should be less than 2mm. For the cost model shown as Eq. (9), the weight coefficients for all the components are assumed equal to each other and therefore they are not considered. In addition, it is assumed that c_{i1} $\left(i = 1, 2, \cdots, L\right)$ is equal to 100 dollars, c_{i2} $\left(i = 1, 2, \cdots, L\right)$ is equal to 1000 dollars and c_{i3} $\left(i = 1, 2, \cdots, L\right)$ is equal to

1.0 for all the parts. Since c_{i1} and c_{i2} are the same for the different parts and their values will not change the optima, they are also not included in the formulations.

In order to illustrate the impacts of the model uncertainty on allocation results, the formulations with and without considering uncertainty are established. The formulation for this example without considering uncertainty is shown as follows:

$$\begin{aligned}
&\min\ f_c\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)=e^{-1.0\left(\frac{T_{11}+T_{12}}{2}\right)}+e^{-1.0\left(\frac{T_{21}+T_{22}+T_{23}+T_{24}}{4}\right)}+e^{-1.0\left(\frac{T_{31}+T_{32}}{2}\right)}\\
&\textit{with respect to}:\\
&\quad T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\\
&\textit{subject to}:\\
&\quad g\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)\le 2\\
&\quad T_{11}-T_{12}=0\\
&\quad T_{21}-T_{22}=0\\
&\quad T_{22}-T_{23}=0\\
&\quad T_{23}-T_{24}=0\\
&\quad T_{31}-T_{32}=0
\end{aligned}\tag{17}$$

where, T_{ij} is optimization variable representing the tolerance of the j^{th} welding point in part i. For example, T_{12} represents the tolerance for the second welding point of part 1. $g\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)\le 2$ represents the constraint that the maximal tolerance of the measurement points in the final product is less than 2mm. The equal constraints in Eq. (17), such as $T_{11}-T_{12}=0$, ensure that the allocated tolerances for the different points at the same part are equal to each other.

In comparison, the optimization formulation for this example considering uncertainty is proposed as follows:

$$\min\ f_c\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)=e^{-1.0\left(\frac{T_{11}+T_{12}}{2}\right)}+e^{-1.0\left(\frac{T_{21}+T_{22}+T_{23}+T_{24}}{4}\right)}+e^{-1.0\left(\frac{T_{31}+T_{32}}{2}\right)}$$

$$\begin{aligned}
&\textit{with respect to}:\\
&\quad T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\\
&\textit{subject to}:\\
&\quad \Pr\left(g'\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)\le 2\right)\ge\left(1-P_r\right)\\
&\quad T_{11}-T_{12}=0\\
&\quad T_{21}-T_{22}=0\\
&\quad T_{22}-T_{23}=0\\
&\quad T_{23}-T_{24}=0\\
&\quad T_{31}-T_{32}=0
\end{aligned}\tag{18}$$

where, everything other than the unequal constraint is the same as in Eq.(17); $\Pr\left(g'\left(T_{11},T_{12},T_{21},T_{22},T_{23},T_{24},T_{31},T_{32}\right)\le 2\right)\ge\left(1-P_r\right)$ represents the constraint that the probability of the maximal tolerance among the measurement points in the final product being less than 2mm is greater than $\left(1-P_r\right)$ considering the model uncertainty. The distribution of the variation simulation output considering uncertainty is assumed as the uniform distribution. These two formulations are implemented using MATLAB® functions.

4.4.3 Results

4.4.3.1 Parametric study of uncertainty. A parametric study is conducted based on the formulations shown as Eq. (17) and Eq. (18) to demonstrate the uncertainty impacts on the allocation results. In this example, the design reliability, $\left(1-P_r\right)$, is assumed as 0.95. Four different values of uncertainty for the variation simulation model are selected shown in Table 4-1. The allocated tolerances for the three parts and the cost for each case are also shown in the table.

Table 4-1. Uncertainty values and the corresponding allocated tolerances

Case #	Uncertainty	Tolerance (mm)			Cost ($)
		Part 1	Part 2	Part 3	
1	0	1.64	1.83	2.17	768
2	0.1	1.53	1.71	1.98	835
3	0.2	1.41	1.62	1.74	917
4	0.3	1.29	1.49	1.59	1004

As shown in Table 4-1, there is only one tolerance for each par in each case even though the tolerances for several welding points in each part need to be allocated. It is because the equal constraints in the formulations shown in Eq. (17) and Eq. (18) force the points on the same part have the same tolerances. Therefore, the allocated tolerances for the both welding points on part 1 are equal to 1.64mm when uncertainty is equal to zero. In the first case, the optimal tolerances are obtained using the formulation in Eq. (17). For the other cases where uncertainty is not equal to zero, the formulation in Eq. (18) is applied.

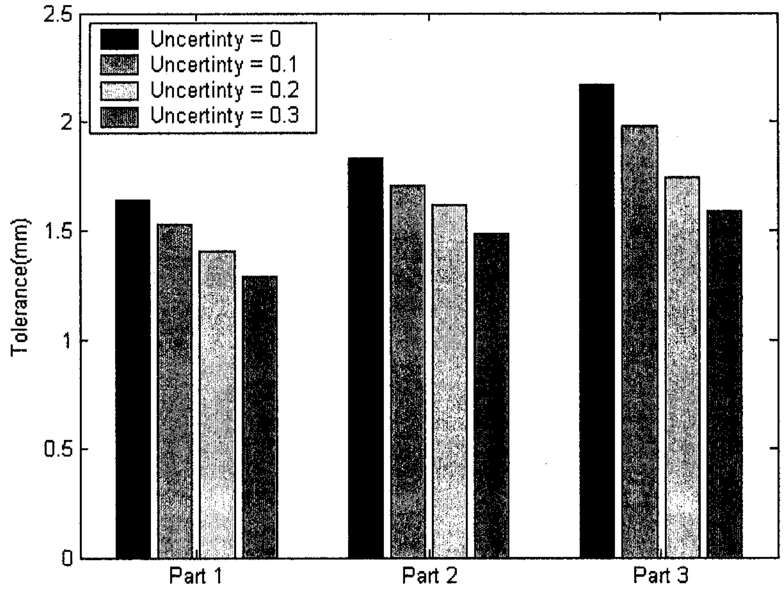

Figure 4-5. Allocated tolerances for parts

Figure 4-5 presents the allocated tolerances for all the parts shown in Table 4-1. As shown in the figure, the tolerances in the first case for each part are the largest among all the four cases. In the other words, the formulation considering uncertainty associated with the simulation models shown in Eq. (18) allocates tighter tolerances to the components than the traditional tolerance allocation formulation shown in Eq. (17) to achieve the same tolerance for the final product. In addition, it can be observed that, for each part, the allocated tolerances are tighter when uncertainty is larger. For instance, while the allocated tolerance for part 1 is 1.53mm when uncertainty is equal to 0.1, the tolerance is 1.41mm when uncertainty is equal to 0.2.

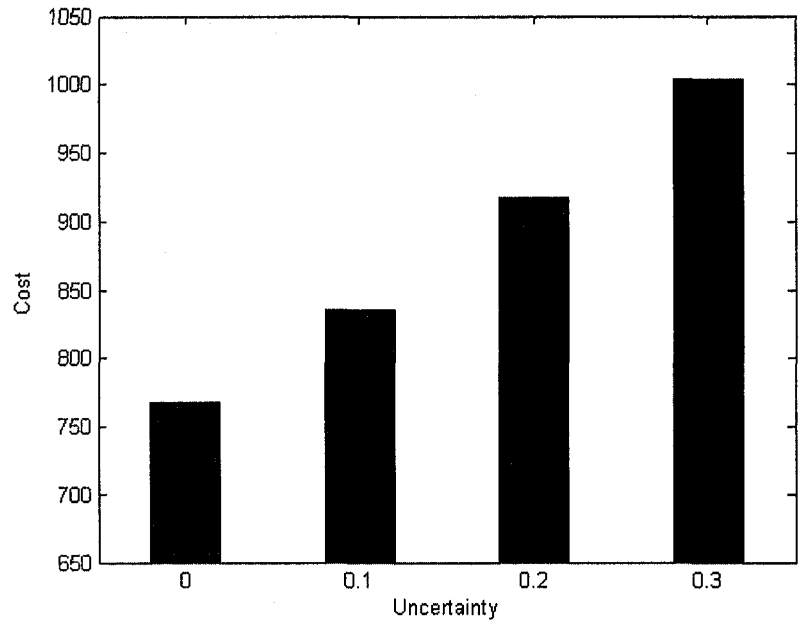

Figure 4-6. Cost vs. uncertainty

Figure 4-6 shows the manufacturing costs of the parts for the different cases. It can be observed that the costs become larger when the uncertainty associated with the simulation models is larger. For instance, the cost increases by about 20 percent when the uncertainty increases from 0.1 to 0.3. The observation is reasonable because the tolerances for the parts are tighter when the uncertainty is larger which was concluded from Figure 4-5, and the tighter tolerances incur the higher cost.

4.4.3.2 Parametric study of the design reliability. Another parametric study is conducted based on the formulations shown as Eq. (18) to demonstrate how the design reliability, $(1-P_r)$, impacts on the allocation results considering uncertainty. In this example, the uncertainty is assumed equal to 0.1. Five different reliabilities are selected shown in

Table 4-2. The allocated tolerances for the three parts and the cost for each case are also shown in the table

Table 4-2. Reliability values and the corresponding allocated tolerances

Case #	Reliability $(1-P_r)$	Tolerance (mm)			Cost ($)
		Part 1	Part 2	Part 3	
1	0.95	1.53	1.71	1.98	835
2	0.75	1.59	1.77	2.04	804
3	0.5	1.64	1.83	2.17	768
4	0.25	1.71	1.92	2.25	733
5	0.05	1.8	2.01	2.45	686

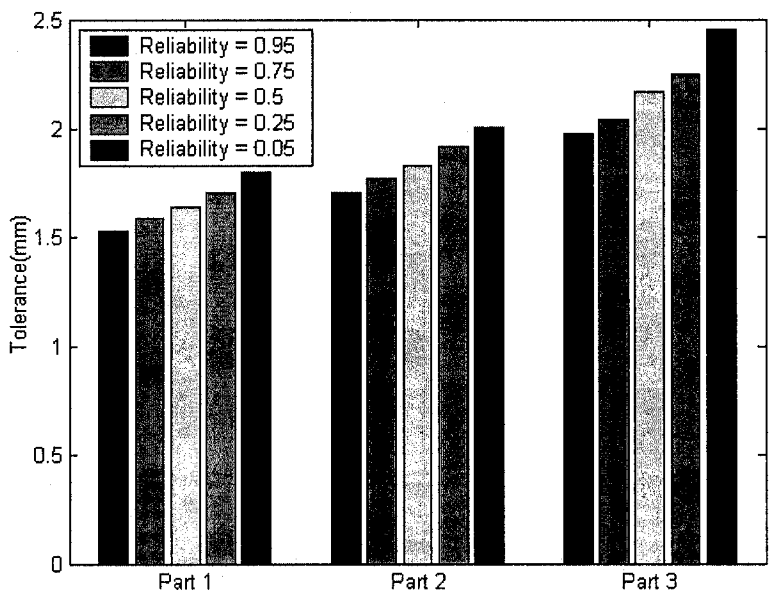

Figure 4-7. Allocated tolerances for parts

Figure 4-7 presents the allocated tolerances for all the parts shown in Table 4-2. It can be seen that the tolerances for each part increase as the design reliability decreases. For example, the allocated tolerance is 1.59mm for part 1 when the design reliability is 0.75. In a comparison, the tolerance is 1.71mm when the design reliability is 0.25. Based on this observation, it can also be implied that the cost decreases when the design reliability decreases, which can be seen from Table 4-2.

4.4.4 Discussion

This case study showed that the uncertainty has the impacts on the allocated tolerances of the components using the proposed formulations shown as Eq. (8). It can be observed that the difference between the traditional and the proposed formulation is the difference between the unequal constraint shown as Eq. (1b) and Eq. (8b) which is arising from the nondeterminacy of the variation simulation model with uncertainty. In addition, the unequal constraint is always active in these two formulations due to the monotone characteristics of the objective function. Therefore, the different optimal results from the two formulations are due to the difference of the unequal constraint.

From the assumption stated before, the prediction of the $g^{'}(T_1, T_2, \cdots, T_i)$ follows the uniform distribution which is shown as follows:

$$g^{'} \sim U\left((1-\zeta_A)g \quad (1+\zeta_A)g\right) \tag{19}$$

where, $g^{'}$ is the model prediction considering uncertainty; g is the model prediction without considering uncertainty; ζ_A is the uncertainty associated with model outputs.

Based on Eq. (19) and the unequal constraint in the formulation considering uncertainty shown as Eq. (8b), it can be derived as follows:

$$\Pr\left(g^{'}\left(T_1,T_2\cdots T_i\right)\leq \hat{T}_A\right)=F\left(\frac{\hat{T}_A-(1-\zeta_A)g}{2\zeta_A g}\right)\geq\left(1-P_r\right) \tag{20}$$

where, $F()$ is the cumulative distribution function for uniform distribution.

From Eq. (20), it can be obtained as:

$$\frac{\hat{T}_A-(1-\zeta_A)g}{2\zeta_A g}\geq\left(1-P_r\right) \tag{21}$$

Then,

$$g\left(T_1,T_2,\cdots,T_i\right)\leq\frac{\hat{T}_A}{\left(1+\left(1-2P_r\right)\zeta_A\right)} \tag{22}$$

Therefore, Eq. (20) is the same as Eq. (22). From Eq. (22), the following three points can be observed:

1) Compared with the unequal constraint in the formulation without considering uncertainty $g\left(T_1,T_2\cdots T_i\right)\leq\hat{T}_A$, it can be seen that these two constraints are the same when the uncertainty ζ_A is equal to zero or P_r is equal to 0.5. This observation is demonstrated by comparing the allocated tolerances shown as case 1 in Table 4-1 where the uncertainty is assumed equal to zero and case 3 in Table 4-2. where the reliability is equal to 0.5.

2) When $(1-2P_r)>0$ the constraint is tighter as uncertainty is larger, which explains the conclusions made in Section 4.4.3.1 where $P_r = 0.05$.

3) When the uncertainty ζ_A is fixed, the constraint is tighter as the design reliability, $1-P_r$, increases. This observation explains the conclusion made in Section 4.4.3.2 that the tolerances are tighter when the design reliability increases.

4.5 Summary and conclusions

In this paper, how the uncertainty associated with a model impacts application of the model was demonstrated through a tolerance allocation example. A general formulation for tolerance allocation considering uncertainty was proposed. A case study was conducted to illustrate the proposed formulation and demonstrate the impacts of the uncertainty on the tolerance allocation. The following conclusions were made and explained:

1) Uncertainty and design reliability have the impacts on the application, tolerance allocation in this paper, of a model.

2) The more accurate model allocates less tight tolerances to the components of a product and therefore less cost of the product.

3) Using a model with some uncertainties, tolerance allocation gives tighter results as higher reliability is required.

Although the conclusions were made with some assumptions, they could be extended to similar models and applications.

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CHAPTER 5

SHAPE REPRESENTATION METHOD FOR VARIATION ANALYSIS OF COMPLIANT ASSEMBLY

Abstract

Research in variation analysis for compliant assemblies has received considerable attention in the last decade. Several methods have been developed to predict how the dimensional variation of components accumulates in such assemblies. However, these methods may generate large discrepancies in the predicted assembly variation when the geometry and variation patterns of the components are complex. These discrepancies are due to simplified consideration in the sources of variation where only the variation at clamping and joining points rather than the whole variation surface of the components is considered. In the cases of complex variation patterns, it was shown that these selected sources of variation are not sufficient to represent the dimensional variation of the components. A shape representation method is proposed in this paper to address this problem. The complex surface variation of the components is decomposed into the variation of an optimal set of key points in terms of their number and locations. In order to locate these points, the variation surface of a component is fit using basic shapes which are determined based on linear mechanics. A more accurate prediction can be obtained by inputting the variation at these key points into the variation analysis models. A case study is conducted to compare the shape representation method with previous methodologies.

The new method significantly reduces the prediction errors of the compliant assembly variation analysis methodologies when parts have complex variation surfaces.

5.1 Introduction

Compliant assemblies are widely used in industries such as automotive, aerospace and home appliances. The compliant elements raise new challenges to understanding their behaviors during assembly. One of the challenges is assembly dimensional variation which can affect functionality, customer satisfaction and cost. For example, automotive bodies with high dimensional variation may cause water leakage, wind noise, big door closing efforts, poor appearance, etc. Therefore, variation analysis for compliant assemblies has been gaining more and more attention.

Several methods to analyze the variation for compliant assemblies have been developed since Takezawa [1980] observed that the traditional addition theorem of variance was not valid for compliant sheet metal assemblies. Liu and Hu [1995] first developed an offset element approach to predict the assembly variation of two beams by combining mechanics models with statistical methods. They extended their research to complex compliant assemblies by using Finite Element Analysis (FEA) [Liu and Hu, 1997]. A linear mechanistic variation model was presented using the method of influence coefficients. The model considered only part variation. Later, Long and Hu [1998] presented a unified model for variation simulation considering part, fixture and welding gun variation. This methodology has been named *C*ompliant *A*ssembly *V*ariation *A*nalysis (*CAVA*). Ceglarek and Shi [1997] and Shiu et al. [1997] developed a tolerance analysis methodology based on the physical/functional modeling for a sheet metal assembly process using flexible beam models. Chang and Gossard [1997] modeled the variation propagation in an assembly process as a contact chain. The contact chain defined the relationship among four concepts: features, variations, displacements and forces. The geometric compatibility, force continuity, and constitutive relations between the nodes in the contact chain were represented by vector equations. However, no analytical equations for the variation propagation in assembly process were presented. Merkley [1998] also developed a linear finite element assembly model. The concepts of material and geometric covariance were proposed and considered in the model.

In summary, these models established a variation relationship between the components and the final assembly. In order to improve the performance of these models, the problem of how to represent the component variation has to be addressed. In variation analysis for rigid assemblies, the variation of a 3D component can be represented by four non-coplanar points which can fully constrain the six degrees of freedom of the component. However, the six degrees of freedom are not always sufficient to represent the variation of a compliant component due to its deformable property. Traditionally, the points to represent the variation of a compliant component are defined as the points where the component will be located and joined [Liu and Hu, 1995,1997]. Therefore, the selection of the points depends only on the specific assembly processes, which is referred to as the process-based method in this paper. In some cases these points will not be able to represent the variation of the whole component. In general, two issues were introduced in the variation analysis due to the lack of variation representation of components: (1) discrepancies between the prediction and the true results; and (2) the same assembly variation may be obtained with the components having different variation shapes, but the same variation at the locating, clamping and joining points.

In order to address these problems and improve the accuracy of compliant assembly variation analysis, this paper proposes a new approach, referred to as shape representation method, to obtain the points needed to represent the variation surfaces of the components. Using the variation at these points instead of just the locating, clamping and welding points as the inputs to the simulation model presented by Liu and Hu [1997], the prediction errors are significantly reduced.

This paper is organized as follows. In Section 5.2, variation analysis method for compliant assemblies proposed by Liu and Hu [1997] is reviewed. An example is used to illustrate the problem associated with the methods. Section 5.3 proposes the methodology to identify the points to represent the variation of components. Section 5.4 conducts a case study to validate the proposed methodologies. Finally, Section 5.5 concludes the paper.

5.2 Review of process-based *CAVA* method

In Liu and Hu [1997], an assembly process was decomposed into four steps which can be illustrated in Figure 5-1 and described as follows:

1) <u>Loading</u> sheet parts to work-holding fixtures; (Figure 5-1(a))
2) <u>Clamping</u> parts to the nominal positions; (Figure 5-1 (b))
3) <u>Joining/welding</u> the sheet parts together; and (Figure 5-1 (c))

4) <u>Releasing</u> the clamps and/or fixtures. (Figure 5-1 (d))

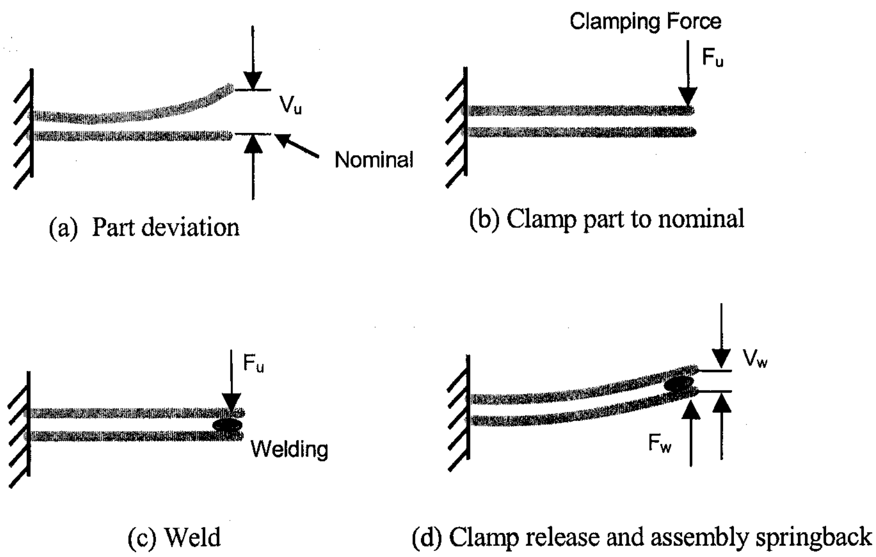


Figure 5-1. Sheet metal assembly process (Liu and Hu [3])

In modeling the assembly variation, a four-step procedure is used:

Step 1: The parts are fixed to the workstation. In the figure, only one source variation is illustrated. If there are more than one source variation, $\{V_u\}$ will be expressed as a vector form.

Step 2: The parts are pushed to the nominal positions by clamping. FEM can be used to calculate the clamping forces:

$$\{F_u\} = [K_u] \times \{V_u\}$$

where:

$\{F_u\}$ is the vector of forces needed to clamp the parts to the nominal position,

$[K_u]$ is the stiffness matrix of the parts from FEM, and

$\{V_u\}$ is the vector of deviation at welding points of parts.

Step 3: After being clamped to the nominal position, the parts are welded together, and the stiffness matrix for the welded structure can be obtained from FEM as well.

Step 4: Fixtures and welding guns are released after the welding, and the assembly springbacks. The springback position can be calculated as:

$$\{V_w\} = [K_w]^{-1}\{F_w\}$$

where:

$\{F_w\}$ is the vector of springback forces,

$[K_w]$ is the stiffness matrix of the assembly from FEM, and

$\{V_w\}$ is the final assembly deviation after springback.

The springback forces are equal to the magnitude of the clamping forces. And, they are being removed after welding. Hence:

$$\{F_u\} = \{F_w\}$$

$$\{V_w\} = [K_w]^{-1} \times [K_u] \times \{V_u\}$$

$$\{V_w\} = [S] \times \{V_u\}$$

where $[S]$ is a matrix which represents how sensitive the assembly deviation is to the deviation of the parts.

As illustrated in Step 2, the forces applied to the source points to push them to their nominal positions were calculated. An implicit assumption is that these forces can push not only these points but also the entire component to the nominal position. In Step 4, as these forces are released, the same amounts are applied in their reverse directions onto the nominal assembly to obtain the springback for the final assembly. However, in some cases the forces pushing the source points to the nominal positions cannot always push the entire component to its nominal position, as shown in Figure 5-2. As a result, simulation errors are induced by applying the springback forces on the nominal assembly to obtain the final shape of the assembly at the step of releasing the clamps and /or fixtures.

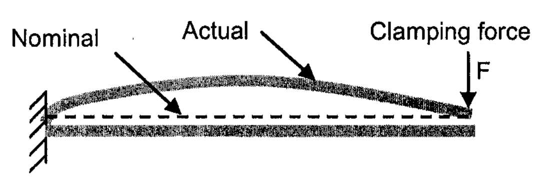
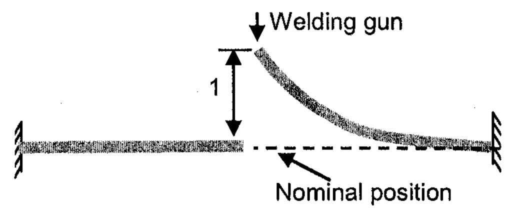
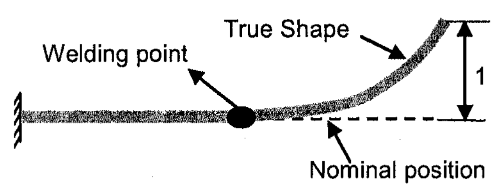


Figure 5-2. Clamped parts before welding

Therefore only applying forces onto these source points is not sufficient to push the whole component to its nominal position except when the part deviation follows the shape of sheet bending. In the other words, the variation at only these source points input to variation analysis models is not sufficient when the part deviation differs from bending deformation. The prediction errors for the assembly will be illustrated in detail by the following example.



(a) Components before assembly



(b) Assembly after springback

Figure 5-3. Assembly model and true results

A simple beam assembly, as shown in Figure 5-3, is used to illustrate the problem. The length of both components in this example is *500 mm*. Using FEM, the parts are represented as structural beams with 20 nodes in each. As shown in Figure 5-3 (a), these two beams are fully constrained. It is assumed that one of the beams is perfect and the other has a unit deviation at the free end where the two beams are to be welded together. The beam with non-nominal shape has a uniform radius of curvature. After releasing the clamping\welding constraints on the imperfect beam, the deviation of the released end of the imperfect component in the final assembly will be equal to the initial deviation of the welded end. In addition, the final shape of the released beam will be a reflection of its initial shape referred to as "True Shape" in Figure 5-3 (b) which is obtained using analytical methods and is the benchmark for the simulation results. As shown in Figure 5-4, the result obtained from the simulation models referred to as "Prediction" which only considered the variation at welding points gives us an incorrect profile of the assembly.

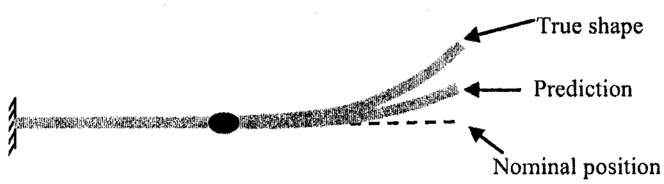


Figure 5-4. True and predicted profiles for final assembly

Another similar example with different shapes for the imperfect component can also be used to illustrate the second issue pointed out in the previous section. The profiles for the imperfect component in the assembly are shown as "Shape 1" and "Shape 2" in Figure 5-5. As shown in the figure, the two imperfect components have different shapes and both have a unit deviation at one end and zero deviation at the other end where the components will be fixtured. As before, these imperfect components are welded with the perfect component together at their free ends. The constraint on the imperfect components is released after welding.

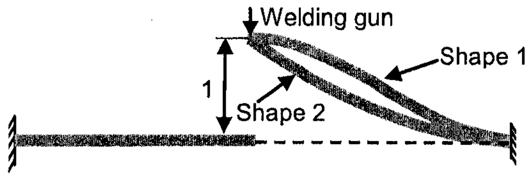


Figure 5-5. Profiles for the imperfect components

The correct assembly profiles are shown as "True 1" and "True 2" respectively in Figure 5-6. As expected, the profiles of the final assemblies for the two cases are different. However, the same prediction profile referred to as "Prediction results for both shapes" is obtained for the two cases using the variation analysis models because the deviations at the welding point for these two imperfect components are the same, which is obviously unacceptable.

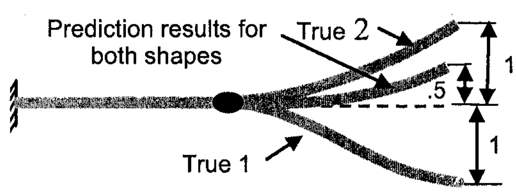


Figure 5-6. Results for the imperfect components

Some discrepancies were shown between the prediction and true results due to insufficient representation of the surface variation. These discrepancies also constrain the application of these models in compliant part design, process simulation, process optimization and so on, where the accurate predictions are needed.

5.3 Shape representation method

The concept of variation shape is introduced and it is defined as the difference between the real and nominal shape of a component. It represents the deviation rather than the real dimensional shape of the component. Therefore, when components are perfect, their variation shapes will be zeros no matter what the nominal shapes of the components are.

The idea of the shape representation method is to determine an optimal set of points at which the variation can represent the variation shape of a component, which assures that the forces to push these points to their nominal positions can push the entire component to its nominal position in the clamping stage. Therefore, the implicit assumption that the entire component is on its nominal position in the clamping stage will be valid. As a result, inputting the variation at these points rather than the source points obtained using the process-based method to the variation analysis models will generate more accurate results.

The points which can represent the variation shape of a component are called the key points of this component in this paper. In order to determine the number and locations of the key points for a component, an algorithm is presented.

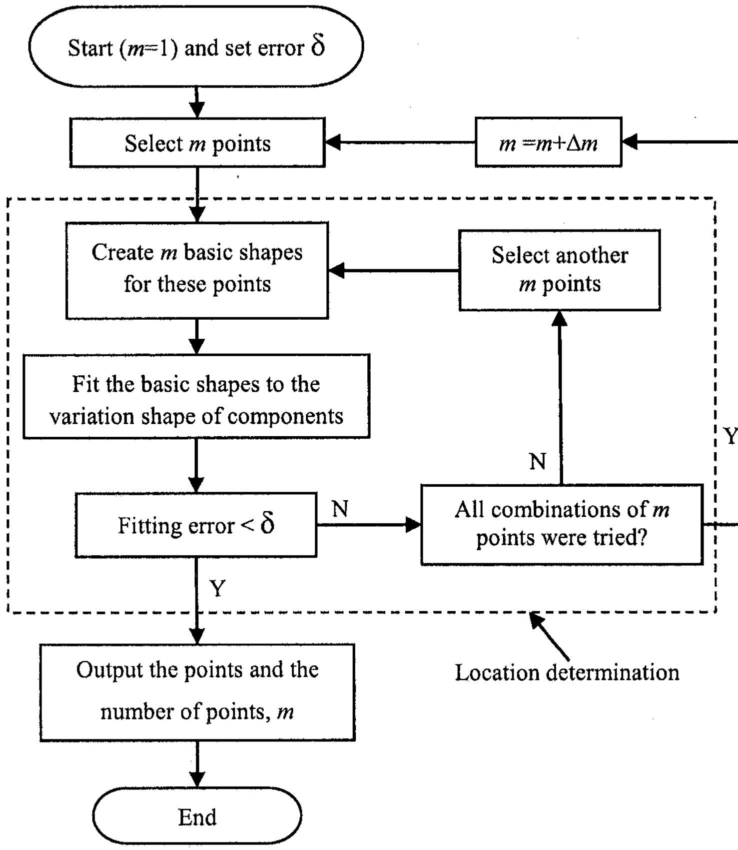


Figure 5-7. Flowchart to determine the key points

Six steps are required to obtain the set of key points for a component.

Step 1: Set $m = 1$ and set the predesignated error δ.

Step 2: Select m arbitrary nodes from the finite element model of the component as the candidates of the key points.

Step 3: Establish the basic shapes for the selected m points.

Step 4: Fit the basic shapes to the variation shape of the component.

Step 5: Check the fitting error of Step 4. If the error is smaller than the predesignated error δ, the candidate points are the key points and m is the number of the key points. Or else, go to Step 6.

Step 6: Check if all the combinations of m nodes have been tried in the finite element model of the component. If so, set $m = m + \Delta m$ (Δm is the step size, it can be one, two or other integer numbers) and go to Step 2. Or else, select another m nodes as the candidates of the key points and go to Step 3.

The complete procedure is illustrated by a flowchart shown in Figure 5-7. In summary, there are two loops in the proposed algorithm: one is for the number of points, m, and the other is for the locations of the points. As a result, an optimal set of points to represent the variation shape of the component can be obtained and the accuracy of the simulation models can be enhanced by inputting the variation at these points into the models.

From the flowchart, it can be seen that the basic shapes play an important role in obtaining the key points for a component. Therefore, the properties of the basic shapes and the algorithm to achieve them will be discussed in the rest of this section.

In order to be used to fit the variation shape of a component and determine a set of points, the basic shapes should have the following properties:

1) A set of basic shapes corresponds to a set of points.

2) In the set of points, each of basic shapes has its own corresponding point where it has a unit deviation. At the other points in the set, it has zero deviations.

3) Basic shapes are dependent on the nominal shape of a component and are independent of the variation shape of the component.

4) Basic shapes satisfy the boundary constraints of the component.

5) Basic shapes of the components are independent of final assemblies.

Because of these properties of the basic shapes, other basic shapes (basic functions) in curve fitting fields, for example, Bezier Curve, Lagrange polynomial, B-Spline and so on, cannot be applied directly to address this problem.

Based on the properties of basic shapes, a set of the nodes in the finite element model of the component can be determined as the key points by establishing a set of basic shapes and fitting the basic shapes to the variation shape of the component. The establishment of the basic shapes will be illustrated by the algorithm depicted as follows:

1) Assume there are m points in the candidates of the key points of a component and the m points are denoted as $\{A_1 \quad A_2 \quad \cdots \quad A_m\}$.

2) Apply a unit force on each of these m points to obtain the compliance matrix for the component. For instance, a unit force is applied on the i^{th} point A_i with the constraints of the component before assembly. Then, the displacements of all these points $\{A_1 \quad A_2 \quad \cdots \quad A_m\}$ due to the force can be obtained as $[v_{1,i} \quad v_{2,i} \quad \cdots \quad v_{m-1,i} \quad v_{m,i}]^T{}_{m\times 1}$. Writing the unit forces and all the displacements as matrices gives:

$$[F]=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}_{m\times m} = [I]_{m\times m}$$

$$[V]=\begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,m-1} & v_{1,m} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,m-1} & v_{2,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{m-1,1} & v_{m-1,2} & \cdots & v_{m-1,m-1} & v_{m-1,m} \\ v_{m,1} & v_{m,2} & \cdots & v_{m-1,m} & v_{m,m} \end{bmatrix}_{m\times m}$$

It is noted that the forces forms an identity matrix. If [*C*] is the compliance matrix and [*K*] is the stiffness matrix, then

$$[V]=[C]\times[F]=[C]\times[I]=[C]=[K]^{-1}$$

3) Invert the compliance matrix to obtain the stiffness matrix. From the above equation, $[K]=[C]^{-1}=[V]^{-1}$ can be derived.

$$[K]=\begin{bmatrix} k_{1,1} & k_{1,2} & \cdots & k_{1,m-1} & k_{1,m} \\ k_{2,1} & k_{2,2} & \cdots & k_{2,m-1} & k_{2,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{m-1,1} & k_{m-1,2} & \cdots & k_{m-1,m-1} & k_{m-1,m} \\ k_{m,1} & k_{m,2} & \cdots & k_{m-1,m} & k_{m,m} \end{bmatrix}_{m\times m}$$

4) Treat the j^{th} column entries in the stiffness matrix [*K*] as forces and apply them to the component on the *m* candidate points with the constraints before assembly. That is, correspondingly apply the forces which are equal to the entries of a vector $\begin{bmatrix} k_{1,j} & k_{2,j} & \cdots & k_{m-1,j} & k_{m,j} \end{bmatrix}^T_{m\times 1}$ onto all the *m* candidate key points $\{A_1 \ A_2 \ \cdots \ A_m\}$ of the component. The displacements of all the nodes of the component under these forces are called a basic shape for the j^{th} point A_j and denoted as:

$$\{U_j\}=\begin{bmatrix} u_{1,j} & u_{2,j} & \cdots & u_{n-1,j} & u_{n,j} \end{bmatrix}^T_{n\times 1}$$

where, *n* is the number of nodes in the finite element model of the component.

5) Run the previous step iteratively to obtain all the m basic shapes for all the points $\{A_1 \quad A_2 \quad \cdots \quad A_m\}$. Finally, they are denoted in a matrix form as $[U] = [\{U_1\} \quad \{U_2\} \quad \cdots \quad \{U_{m-1}\} \quad \{U_m\}]_{n\times m}$.

In order to illustrate the algorithm, a set of basic shapes of the imperfect component shown as "Shape 1" in Figure 5-5 is obtained. The finite element model of the component is shown in Figure 5-8, where the X axis is the node number and Y axis is the deviation for each node. In this illustration, the nodes (*1, 5, 9, 13* and *17*) are arbitrarily chosen as the candidate points ($m = 5$). One of the five basic shapes obtained by this algorithm is shown in Figure 5-9. As shown in the figure, this basic shape has a unit deviation at its corresponding point *5* and zero deviations at other points (*1, 9, 13* and *17*). Moreover, this basic shape has zero deviation at the right end of this component because the component is fixtured at that end before assembly.

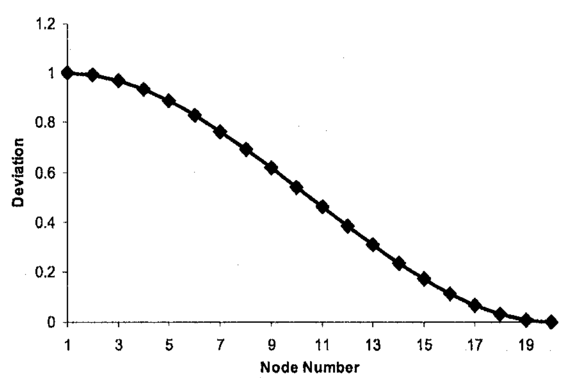


Figure 5-8. Variation shape of component

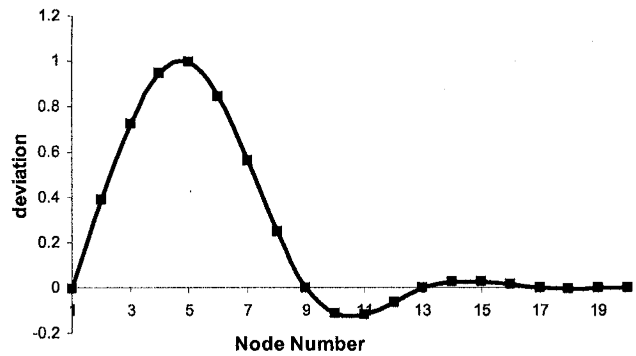


Figure 5-9. One of basic shapes for component

5.4 Case study

In this section, the proposed shape representation approach will be validated by the example shown in Figure 5-5 with the imperfect component of "Shape 1". In this example, the material for the components is steel with Young's module $E = 3.06\ GPa$ and Poisson ratio $\nu = 0.3$. The lengths and the thicknesses of these two components are equal to *500 mm* and *5 mm,* respectively. The element type used in finite element models for these components is the beam element. These finite element models are built using Hypermesh® 4.0. The FEA software used in this example is MSC.Nastran®.

Firstly, several sets of key points with different number of points are obtained using the proposed method. Corresponding to these sets of key points, there are different sets of basic shapes. In Figure 5-10, the logarithmic fitting errors of these sets of basic shapes to the variation shape of the component are shown as "fitting error". In the figure, X axis is the number of the key points. Meanwhile, the prediction results of the assembly variation are obtained using the variation at the sets of key points as input to the variation analysis models. Prediction errors can be obtained by comparing the prediction results and the true result which was shown as "True 1" in Figure 5-6. The logarithmic prediction errors are shown as "prediction error" in Figure 5-10.

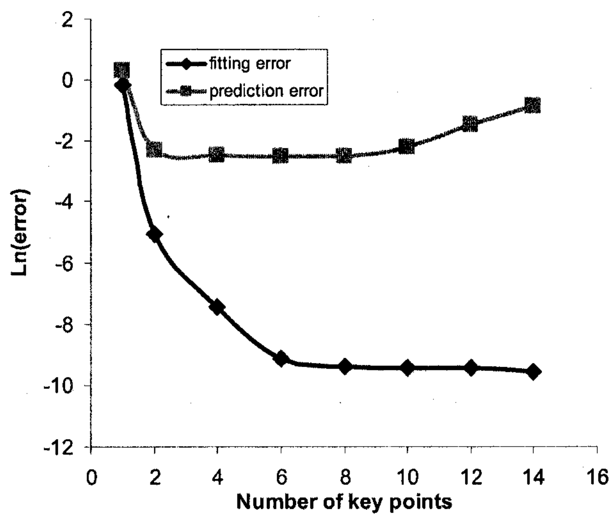


Figure 5-10. Fitting and prediction errors for different sets of key points

As can be seen from the figure, the fitting errors are always decreasing as the number of key points increases. The prediction errors are also decreasing with the number of the key points increasing until the number is greater than eight. After that, the prediction errors increase as shown for points "10, 12, 14" in the figure. The reason is when the number of the key points is small, the variation shape of the component will be better fit as the number of the key points increases. Therefore, the prediction of the simulation model becomes better using the corresponding key points. However, the computational errors of the simulation model will also increase as the number of the key points increases, which explains the increases of prediction errors when the number of the key points is greater than eight.

In order to show the improvement using the proposed method compared with the traditional simulation model, the case of "8" in Figure 5-10 is illustrated in detail. Using the proposed approach eight key points (*1, 3, 5, 8, 13, 16, 17*, and *18*) are obtained which are selected to represent the variation shape of the imperfect component. Figure 5-11 shows the result obtained from *CAVA*, the true result and the result of this proposed approach which is called "Shape Representation". Only the deformation for the imperfect component is shown because there is no deformation for the perfect component in the final assembly. As shown in Figure 5-11, the discrepancy of the *CAVA* result is much bigger than that of proposed method compared with the true result. Therefore, this approach improves the prediction and simulation accuracy of the traditional variation analysis models.

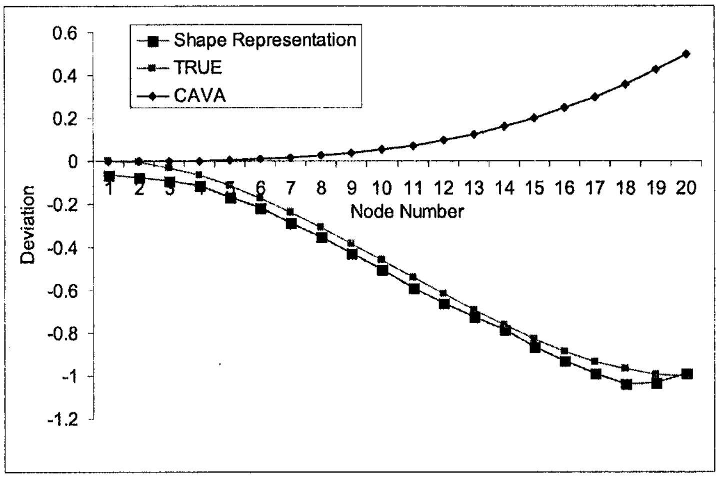


Figure 5-11. Comparison of the results

From the result of this case study, it can be concluded that:

1) An effective method to determine the key points to represent the variation shape of a component was developed, and

2) Shape representation method is a promising approach to improve the prediction accuracy of variation analysis for compliant assemblies. The prediction errors are mitigated significantly by this approach, at least for this example.

5.5 Conclusions

A shape representation method has been developed to identify the key points needed to represent the components variation in compliant assembly variation analysis. In order to obtain these points, an algorithm was developed to decompose the variation shapes of components into the basic shapes which could be acquired through the proposed methodology in this paper. Finally, the case study showed that the shape representation method effectively and significantly reduce the prediction error of variation analysis models.

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CHAPTER 6

SUMMARY, CONCLUSIONS AND FUTURE WORK

6.1 Summary and conclusions

In this dissertation, research on several topics has been performed based on the variation simulation models for multi-stage manufacturing systems.

In Chapter 2, pattern sensitivity, component sensitivity and station sensitivity indices have been defined based on compliant assembly variation simulation models. These three indices can be utilized for measuring the sensitivity of a product dimensional quality to the variation of a pattern, an individual component and the components assembled at a particular station, respectively. In addition, a method has also been proposed to obtain the ranges, the minimum and the maximum, for all the indices which can be used to estimate sensitivities without any information about incoming variation. In other words, the estimation of sensitivities is independent of the input variation. This independence is necessary and helpful in most cases at the design stage when limited information for the components variation is available. At the end of the chapter, a case study has been conducted to evaluate the definitions of these sensitivities.

In Chapter 3, the characteristics and the sources of the uncertainty for the simulation models of a multi-stage manufacturing system have been analyzed. In order to analyze the propagation and accumulation of the uncertainty sources, an uncertainty model based on a state space model has been developed. Based on the uncertainty model, the guidelines for model calibration were also established. From this research, several conclusions were drawn: 1) uncertainties will propagate and accumulate to simulation results while running a state space model along the stations in a multi-stage manufacturing system. Therefore, the accuracy and fidelity of the simulation results is reduced; 2) the output uncertainty of a station model which is related to the uncertainties associated with the state matrix, the input matrix, the state vector and the input vector plays a critical role in the output uncertainty of the state space model; and 3) the uncertainties for incoming parts and fixtures, do induce some uncertainties but not propagate or accumulate to the results when simulation models run along the stations of a system.

In Chapter 4, an optimization formulation for tolerance allocation considering the uncertainties of variation simulation models has been proposed. A case study was conducted to illustrate the proposed formulation and demonstrate the impacts of the uncertainty on tolerance allocation. The following conclusions were made: 1) the uncertainties and design reliability have impacts on tolerance allocation; 2) the less tight tolerances are allocated to the components of a product using the models with less uncertainties; and 3) Using a model with some uncertainties, tighter tolerances are allocated as more reliability is required.

In Chapter 5, a shape representation method has been developed to identify the key points needed to characterize the surface dimensional variation of a component in compliant assembly variation simulations. In order to obtain these points, an algorithm has been developed to decompose the surface variation of a component into basic shapes which could be acquired through the proposed methodology in this research. A case study showed that the shape representation method could effectively and significantly reduce the prediction errors therefore the uncertainty of the simulation models.

6.2 Contributions

The following contributions to the variation simulation models for multi-stage manufacturing systems have been achieved by this research:

1) A set of sensitivity metrics has been defined. They can be effectively used in the evaluation of the variation sensitivity of a compliant assembly to its components' variation from different aspects. The formulations proposed in the research could be extended to the variation simulation models for other manufacturing systems such as rigid assembly and machining systems,

2) The uncertainty for multi-stage variation simulation models has been explored. A formulation for the quantification of uncertainty was proposed. An uncertainty model based on variation simulation models in the state space form has been developed. Based on the uncertainty model, the uncertainty propagation and accumulation of uncertainty in multi-stage variation simulation models can be analyzed, the impacts of uncertainty on the simulation results can be evaluated, and the guidelines for the calibration of variation simulation models can be established,

3) An optimization formulation for tolerance allocation has been proposed considering uncertainty of variation simulation models. The necessarity of considering uncertainty has been explained and the impacts of uncertainty on tolerance allocation results have been analyzed. The ideas and methodologies in this research can be extended from tolerance allocation to other applications of variation simulation models, and

4) A part representation method to consider the complex surface shape of incoming parts in compliant variation simulation has been developed. This method has been employed for identifying the key points where the variation is inputted to the variation simulation model for lower uncertainty.

6.3 Future work

The methodologies and models proposed in this research could be further improved and /or extended in the following directions:

1) To provide designers with a better overview for the variation impacts of parts to final assembly dimensional quality, contribution analysis besides the sensitivity analysis of the variation of incoming parts should be conducted,

2) To demonstrate the propagation and accumulation characteristics of different uncertainty sources in a multi-stage variation simulation model, the sensitivity for the different sources should be analyzed,

3) Guidelines and methodologies for estimating the uncertainty of a model should be established and developed,

4) Due to the fast-growing applications of variation models in manufacturing areas and the increasing importance of simulation results to designers and engineers, a model for evaluating the risk effects incurred by uncertainty needs to be developed and guidelines to manage and reduce the risk in the applications of variation simulation models is worth of being established,

5) Since the part shape representation method is computationally intensive and time consuming due to the utilization of Finite Element Method (FEM) and genetic algorithm (GA), a more efficient and realistic method has to be developed, and

6) Because only several points on a surface of a part may be measured in reality and however the whole surface dimensional variation information is necessary for a more accurate result in compliant variation simulation models, an algorithm needs be developed to estimate the surface variation from the measurements of several points on the surface.

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